ELEMENTARY MATHEMATICS METHODS COURSES AND BEGINNING MATHEMATICS TEACHERS' READINESS TO TEACH NUMBERS AND OPERATIONS

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ABSTRACT

Elementary student achievement in the area of mathematics is a subject of educational, political, and economic studies and reform movements. Research showed overall low achievement in mathematics. As teachers play a crucial role in student achievement (Hattie, 2009), it is important that educational research include the study of teacher preparation. Research shared in this study indicated mathematics methods courses can positively affect preservice teachers' mathematical attitudes and learning of effective instructional practices in mathematics. Despite these positive influences, many first-year elementary teachers do not translate the philosophies, beliefs, conceptual knowledge, and pedagogies learned in their elementary teacher preparation program courses into their first-year teaching practices.

This study investigated the extent to which eight teachers felt their attitudes and confidence, mathematical content knowledge, and mathematical pedagogical knowledge were influenced by their mathematics methods coursework. The study also investigated whether these beginning teachers' perceptions of readiness changed over the course of their first semester of teaching, and includes self-reported details about why and how the perceptions changed.

This study provided an opportunity to understand the experiences of first-year elementary mathematics teachers. Findings suggested that teachers perceived themselves to be positively affected by their methods coursework in the area of attitude, and all teachers were encouraged to use constructivist pedagogies. The majority of teachers felt their content knowledge was not greatly affected by their methods courses. While the teachers indicated examples where they had implemented some learning from their methods courses, findings also seemed to indicate examples of instances where teachers' choices had been influenced more by their school's or district's context than their preparation program. Recommendations for future mathematics methods courses, based on these teachers' feedback and included research, are discussed.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1. INTRODUCTION

1.1 Need for the Study

Elementary students' achievement in the area of mathematics is a subject of educational, political, and economic studies and reform movements. Studies show overall low achievement in mathematics. The Program for International Student Assessment (PISA), given internationally every three years, was most recently administered in 2015 to students in 73 countries. The National Center for Educational Statistics (2015) website reported that the U.S. average PISA score was 470, lower than the overall average of 490 among all participating countries. The U.S.'s PISA average mathematics score in 2015 was also lower than the average scores in 2012 and 2009.

The Trends in International Mathematics and Science Study (2015) assessment ranked the United States 14th out of 49 participating countries. The National Assessment of Educational Progress (2017) showed the U.S. mathematics achievement average scale score for fourth graders was a 240 out of 500 with only 40% of students scoring at or above the level of proficient.

These assessment results appear to suggest that improvements need to be made in our country's offerings of mathematics education. Hattie's (2009) meta-analysis of educational research suggests that teachers play a crucial role in student achievement. Therefore, this study focuses on elementary teacher preparation programs in general, and elementary mathematics methods courses in particular to improve educational practices in classrooms by improving the mathematical attitudes, content knowledge, and pedagogical practices of teacher candidates so they are prepared to effectively teach mathematics even in their first year of teaching. This study will specifically look at how incorporating growth mindset and constructivism into elementary mathematics methods courses affects beginning teachers' perceptions of preparedness to teach number and operations concepts.

Previous studies have focused on analyzing the mathematics methods courses offered to elementary preservice teachers. Greenberg and Walsh (2008) examined 257 syllabi and required texts from 77 undergraduate elementary education programs (p. 3). Only ten schools in the sample scored high on both the course work and textbook quality, and 35 schools failed on both syllabi and required textbook examination (p. 32).

Greenberg and Walsh (2008) looked at college requirements for teacher preparation programs in terms of the type and number of mathematics and mathematics methods courses necessary for degree completion. Of the 33 education schools reviewed, four did not require a mathematics methods course, 15 had mixed methods courses combining other subject areas or mathematics content, 12 offered a single methods course for elementary and middle school mathematics educators, and one offered a course combining mathematics, science and technology for elementary and middle school educators (p. 44).

Two-thirds of the education schools reviewed use mathematics textbooks deemed inadequate by the reviewers in their study for coverage of the major topics in mathematics (p. 35). Of the two most widely listed required texts, one received the lowest score possible for coverage of number and operations which makes up the "majority of the foundational mathematics taught at the elementary level" (p. 36).

The authors cited data published by the College Board stating that in 2008 college bound seniors who planned to major in education had an average SAT mathematics score of 483, well below the national average of 515 for all college bound seniors. In 2016, the College Board's Total Group Profile Report for the SAT generated a list of 36 intended college majors. The average mathematics score for students reporting they were education majors was lower than 25 of the other listed majors. These statistics suggest that students going into elementary education may have lower mathematics abilities and possibly negative attitudes towards mathematics.

Startz (2017) looked at which colleges train teachers and how selective those colleges were in admitting students. Education majors were more likely found at less selective colleges, relatively few came from large, "flagship" universities, and more education majors came from regional colleges with lower bars for entry (p. 2). His results suggest that graduates with education degrees are disproportionately found at schools where students have lower SAT score requirements and scores.

Greenberg and Walsh (2008) reported that mathematical content is frequently missing or weak in the coursework of elementary mathematics methods courses (p. 43). Many of these courses did not include practice teaching opportunities either. While most elementary mathematics content courses were taught in the mathematics department, many times the instructor was the newest faculty or the faculty member who had to "take a turn" (p. 46). Mathematics course work for elementary preservice teachers were not demanding in content or expectations (p. 47). The authors questioned the rigor of the assignments and assessment items on tests shared for review. Because most schools use the Praxis or state specific licensure tests upon degree completion, preservice teachers can pass without demonstrating proficiency in mathematics or other subjects "which makes it impossible to know how much mathematics elementary teachers know at the conclusion of their teacher preparation" (p. 41).

Greenberg and Walsh (2008) noted that many preservice teacher educators worry that there is not enough time to cover all important mathematics topics in one course. On that note, they included a statement from the study's Mathematics Advisory Group that reads:

> …we strongly recommend teacher candidates take a minimum of three mathematics courses designed specifically for prospective elementary teachers which deal explicitly with elementary and middle school topics. This coursework should be coupled with one mathematics methods course (p. i).

They suggested that preservice teachers need a heavy focus on number and operations without eliminating coverage of other strands. "Instructors should not provide the same kind of mathematics education that the preservice teachers had as a child with more procedural knowledge of mathematics" (p. 28). Nor should it be "dumbed down" or crammed with mathematics concepts applicable to too wide a range of grades (p. 55).

These findings seemed to suggest that education majors, on average, may have lower mathematics abilities, scores, and requirements over the years. This may suggest that many first year teachers are entering the elementary classroom unprepared to teach mathematics concepts. Unfortunately, research (Beckman, Wells, Gabrosek, Billings, Aboufadel, & Curtiss, 2004) has found that weaknesses in the teachers' knowledge of the mathematics concepts contributed to lower mathematics achievement in their students. Teacher weakness in mathematics was found to co-exist with high levels of teacher-held math anxiety. Therefore the mathematics methods courses carry a large responsibility when preparing teachers for effective elementary mathematics instruction. The preservice teachers most likely will need different levels of support in developing positive mathematical attitudes, content knowledge, and pedagogy specific to mathematics instruction. Details about research in these three areas are included in this review of literature to highlight research showing positive effects mathematics methods courses can have on preservice teachers.

1.2 Statement of the Problem

Cochran-Smith and Villegas (2014) and Darling-Hammond (2010) have studied teacher preparation programs, certification programs, licencing requirements, and pathways to teaching careers among different countries and within the U.S. They have also studied the student achievement connected to each of these teacher variables to determine the effectiveness of different types of teacher preparation programs and pathways. Cochran-Smith and Villegas (2014) found that students in classes whose teachers graduated from a university teacher preparation program achieved greater gains in mathematics than students whose teachers completed programs with reduced coursework or as compared to similar expectations of the Teach for America program. Overall, the researchers found that there is great variability in teacher preparation programs and pathways and their effectiveness. In fact, the authors noted the "variation in effectiveness was much greater *within* than *between* pathways" (p. 16).

Darling-Hammond's (2010) research summarized features of effective programs such as student teaching experiences, amount of coursework in reading and mathematics, study of local curriculum, percentage of tenure-line faculty providing educational courses, and coursework "helping candidates to learn to use specific practices and tools that are then applied to their clinical experiences" (p. 40). She also noted the great variability among teacher preparation programs, requirements, and pathways. Her advice was that "we need highly effective, adequately resourced models of preparation for all teachers, without exception" (p. 39).

Maxwell (2014) compared teaching practices of traditional versus alternatively prepared teachers. His study included a history of alternative certification of teachers in the U.S. He included 2005 data from the National Center for Education Information (NCEI) which reported that there were "115 alternative routes to teacher certification being utilized in 43 states and the District of Columbia" (p. 7). Maxwell noted research from Mohr (2006) that many teacher education programs were modifying their programs to compete with alternative programs that provide a shorter, faster track to a teaching degree or certificate. It may be that the large number of alternative routes to teaching and teacher preparation programs' responses to the programs helps explain why there could be variation in the teacher workforce.

Kastberg, Sanchez, Tyminski, Lischka, and Lim (2013) suggested ways to reduce variability in the preparation of mathematics teachers. They stated that mathematics teacher educators should come together to study and share "the development and enactments of activities, with attention to frameworks, contexts (such as program size, student population, characteristics of space), critical decision points, as well as impacts" (p. 1355). Effective mathematics methods coursework curriculum and practice "would allow for replication by other mathematics teacher educators wishing to foster similar outcomes" (p. 1351). This is not to say that every teacher preparation program must be exactly the same as there are cultural and environmental factors to be considered in teacher preparation as well. However, certain research-based practices in teacher preparation programs such as positive, carefully chosen student teaching experiences and other characteristics to be discussed in this study's literature review should be consistently utilized in planning mathematics methods course.

As mentioned, there is research that mathematics methods courses positively affect preservice teachers' mathematical attitudes and learning of effective instructional practices in mathematics. The Association of Mathematics Teacher Educators (AMTE) published standards for preparing elementary preservice teachers in 2017 based on this research. Numerous projects surveyed preservice teachers about their levels of math anxiety, attitudes about mathematics, confidence levels in teaching mathematics, basic procedural and deeper conceptual mathematical content knowledge, general pedagogical knowledge, and specific mathematics pedagogical knowledge. These surveys were typically conducted at the beginning and end of the semester long method course to see if the methods courses had a positive effect on the preservice teachers and, therefore, could hope to improve the quality of the teacher preparation program offerings.

Despite these positive influences and standards, many first year teachers do not translate the philosophies, beliefs, conceptual knowledge, and pedagogies learned in their teacher preparation program courses into their first year teaching practices (Cochran-Smith, Villegas, Abrams, Chavez-Moreno, Mills, & Stern, 2015; Darling-Hammond, 2010; Harris, 1991; Kastberg et al., 2013; Quinn, 1998; Smith & Avetisian, 2011; Valli, Raths, & Rennert-Ariev, 2001). Cochran-Smith and colleagues (2015) noted that "novice teachers struggle with the reality of schools" (p. 113). Therefore, there is a need to study first year teachers to determine what barriers exist during the first year of teaching that may limit or entirely prevent teachers from applying what they learned and experienced in their mathematics methods courses.

Beginning teachers' perceptions of readiness and personal experiences can shed light on their own strengths and provide insight into how their weaker areas could be refined. This is not to say that the strengths and weaknesses must be a result of a teacher preparation program, a particular mathematics methods course, or their methods course instructor. However, mathematics methods instructors in teacher preparation programs could still benefit from this first year teacher feedback to improve program curriculum, readings, and assignments to combat the potential barriers faced by beginning teachers.

1.3 Purpose of the Study

The purpose of this study was to investigate the perceptions of readiness to teach number and operations concepts among first year teachers who took mathematics methods coursework in their teacher preparation program. This study investigated the extent to which the beginning teacher felt their attitudes and confidence, mathematical content knowledge, and mathematical pedagogical knowledge were influenced by their mathematics methods coursework. The study also investigated whether these beginning teachers' perceptions of readiness changed over the course of their first semester of teaching, and includes self-reported details about why and how the perceptions changed.

1.4 Research Questions

These research questions guided the examination of beginning teachers' perceptions of their mathematics methods course(s), and how those perceptions may have changed over the course of the first semester of teaching elementary mathematics particularly in the area of number and operations. Each question addressed perceptions in the same three categories of attitude and confidence, mathematical content knowledge, and pedagogical knowledge.

In each of these three sections, the focus was narrowed further to specific aspects of teacher preparation that this researcher sought to understand better. First, the researcher wanted to focus on first year teachers' attitudes and anxiety in mathematics to determine the extent to which they were exposed to growth mindset in their methods courses. This researcher theorized that that exposure might positively affect attitude. It was hoped that study participants would share evidence of growth mindset in their classroom practices.

Next, this researcher wanted to focus on how methods courses utilize constructivist methods to affect teacher preparation in the areas of content and pedagogical knowledge. Interview questions were designed to determine the extent to which participating teachers experienced these methods and what effect it had on their Mathematical Knowledge for Teaching.

I. What are beginning elementary teachers' perceptions of how well their mathematics methods course(s) prepared them for their first year of teaching number and operations?

A. **Attitude and Confidence**

Do beginning teachers show evidence of a growth mindset? What are beginning teachers' ratings of confidence in their ability to teach concepts in number and operation?

B. **Mathematical Content**

With regard to the topic of number and operations taught or reviewed in college mathematics methods courses, what are beginning teachers' perceptions of the quality of mathematical content presented in terms of how much time was devoted to these topics and the depth of exploration into the concepts?

C. **Pedagogy**

How were beginning teachers exposed to constructivist pedagogies in their mathematics methods course(s)? What are beginning teachers' perceptions of their experiences with constructivist pedagogies? Which of these pedagogies learned in their mathematics methods course(s) will they apply in their own lessons on number and operations?

II. What changes, if any, are there in beginning elementary teachers' perceptions of how well their mathematics methods course(s) prepared them for their first year of teaching

number and operations?

A. **Attitudes and Confidence**

Do beginning teachers continue to show evidence of a growth mindset during the first year of teaching? What changes, if any, do beginning teachers have in their mindset and ratings of confidence during their first year of teaching?

B. **Mathematical Content**

During their first year of teaching, what changes, if any, are there in beginning teachers' perceptions of the quality and quantity of mathematical content in the area of number and operations taught or reviewed as part of their college mathematics methods curriculum(s)?

C. **Pedagogy**

Which constructivist pedagogies learned in their mathematics methods course(s) did they apply in their own lessons on number and operations?

CHAPTER 2. REVIEW OF THE LITERATURE

In this paper, a variety of research describing mathematical methods courses were reviewed in order to highlight the positive effect mathematics methods course(s) can have on preservice teachers in the areas of attitude and confidence, mathematical content knowledge, and mathematical pedagogical knowledge. This chapter contains research on how growth mindset, constructivism, and Mathematical Knowledge for Teaching (MKT) may be applied in mathematics instruction for both the elementary preservice teachers during their mathematics methods coursework, and for their future students. Specific constructivist pedagogies like manipulatives, micro-teaching, mathematical modeling tasks, inquiry-based lessons, and games are included to give examples of pedagogies which can be utilized in mathematics methods courses and elementary mathematics classes. The final section of this literature review focuses on how and why the positive effects from mathematics methods courses may or may not translate into first year teaching practices. Possible barriers to applying teacher preparation program learning are also discussed.

2.1 Attitudes and Confidence

The Association of Mathematics Teacher Educators (2017) specifically included "positive attitudes" and "productive dispositions towards teaching and learning of mathematics" (p. 53) as essential standards for well-prepared beginning teachers. Research has shown that mathematics methods courses have a positive impact on preservice teachers' attitudes (Ball, 2009; Bekdemir, 2010; Bursal & Paznokas, 2006; Lomas, 2009; Peker, 2009; Quinn, 1997; Quinn, 1998; Robinson & Adkins, 2002; Saran & Gujarati, 2013; Swars, 2005) and confidence in teaching mathematics (Althauser, 2018; Ball, 1990; Conrad & Tracy, 1992). Specific aspects of preservice teachers' attitudes include mathematics teacher efficacy, identity, sociomathematical authority, and math anxiety. Each of these areas was found to affect teachers' decision-making and classroom practices (Bursal & Paznokas, 2006; Charalambous, 2010; Lomas, 2009) such as how much time was scheduled for mathematics, and what types of tasks and activities students were assigned. Althauser (2018) defined teacher *self-efficacy* as the "extent to which a teacher feels capable to promote student learning in any content area" (p. 55). Unlu (2018) stated that teachers with low self-efficacy beliefs in mathematics avoid challenging tasks and activities.

Swars (2005) defined *teacher efficacy* as the teacher's belief in his/her own skills and ability to be an effective teacher. Swars' study included data from four elementary preservice teachers enrolled in a three-credit hour mathematics methods course. It included 24 days of clinical experience in elementary classrooms. The preservice teachers completed the Mathematics Teaching Efficacy Beliefs Instrument (Enochs, Smith, & Huinker, 2000), a two-part survey that gathered data about their degree of mathematics teaching efficacy. The study found highly efficacious teachers were more likely to use inquiry and student-centered teaching strategies, possessed lower math anxiety, and more likely to try new teaching strategies.

Saran and Gujarati (2013) conducted a study to determine the effects of a mathematics methods course on teachers with low efficacy, negative beliefs, and negative attitudes towards mathematics in general. The methods course was specifically designed to follow the Concrete-Pictorial-Abstract (CPA) instructional methodology suggested by Jerome Bruner in 1966. Qualitative data collected from reflective narratives and interviews of 145 preservice teachers over a two-year period revealed that the majority entered the course with negative beliefs about their own mathematical identities. The authors defined mathematics identity by saying that it "encompasses an individual's self-perception of their knowledge of mathematics, confidence level to teach mathematics, and beliefs about their mathematics teaching competencies" (p. 106).

The majority of participants (Saran & Gujarati, 2013) reported an increase in their comfort and confidence levels in mathematics by the end of the course. They attributed this increase to a deeper understanding of the concepts themselves. The participants changed their beliefs about mathematics, noting that conceptual understanding was more important than procedural knowledge. They understood how to use manipulatives, exploration, discovery, and various strategies to solve problems.

Saran and Gujarati (2013) used the term *sociomathematical authority* as "mathematical dispositions and a sense of intellectual autonomy in mathematics" (p. 101). The authors stated that teachers' beliefs influence their perceptions and motivation for teaching mathematics effectively. Exposure to the CPA method and constructivist methods in the course resulted in reports from participants that they felt "prepared to teach content when they assume their own classrooms since they now had a much greater conceptual understanding of mathematics" and they no longer felt "intimidated about teaching mathematics" (p. 109).

2.1.1 Math Anxiety

Math anxiety is one piece affecting mathematics identity and sociomathematical authority. Bekdemir (2010) described math anxiety as "an illogical feeling of panic, embarrassment, flurry, avoidance, failing and fear, which are physically visible, and which prevent solutions, learning and success about mathematics" (p. 312). The concept of math anxiety was first introduced by Mary Fides Gough (1953) as mathemaphobia to try to explain why students were failing her mathematics class despite being proficient in other subjects. Dreger and Aiken (1957) noted that emotional factors can hinder mastery of mathematics concepts. They would go on to publish studies on attitudes towards mathematics and, by the early 1960s, would develop a mathematics attitude scale using Likert style questions. Finally, Richardson and Suinn (1972) published the Mathematics Anxiety Rating Scale (MARS), which utilized 98 Likert style items. Even after all this time, math anxiety is still a problem. In fact, Peker (2009) noted the term mathematics teacher anxiety has now been used since the 1990s.

Hadfield and McNeil (1994) stated that math anxiety causes could be categorized as environmental, intellectual, and personality factors. Environmental factors are classroom, teacher, or parent based. Intellectual factors included negative attitude, lack of confidence in mathematics ability, or low persistence levels. Personality factors include such things as reluctance to ask questions because of shyness, lack of self-respect, and gender bias. Learning styles are another personality factor that affect math anxiety (Peker, 2009).

Bekdemir (2010) conducted a study of 167 preservice teachers during their senior year of college to examine whether their worst and most troublesome experiences in a mathematics classroom caused their math anxiety. The Mathematics Anxiety Scale and the Worst Experience and Most Troublesome Mathematics Classroom Experience Reflection Test were administered to all participants. The ten highest scoring preservice teachers were interviewed and data were analyzed using thematic coding. Unfortunately, he found the majority of their worst experiences and anxiety were caused by environmental factors, namely their own teachers' behaviors or teaching approaches. This study matches Brown et al. (1999) who found that the mathematical understanding of preservice teachers was influenced by a "strongly affective account of their own mathematical experiences in schools, where mathematics was often seen as difficult and threatening" (p. 299). Bekdemir (2010) described the danger of forming a math anxiety cycle when mathematically anxious teachers "transfer that anxiety to their students" (p. 313) either through hostile, inadequate, or apathetic behaviors towards mathematics or towards the teaching of mathematics.

In summary, methods course instructors have a daunting task of creating a positive, yet challenging course. Preservice teachers may already suffer from varying levels of math anxiety. In the next section, a review of growth mindset and mathematical mindset research includes a description of how mindset instruction can be applied to mathematics education for preservice teachers and elementary students to combat negative attitudes and math anxiety. Critiques and concerns to consider during the implementation of growth mindset in mathematics education is also addressed.

2.1.2 Growth Mindset in Mathematics Education and Teacher Preparation

Many elementary schools have begun to encourage a growth mindset as a way to improve student attitudes and achievement. In this section, the work of Dweck (2006), Ricci (2015), Boaler (2016), and Nottingham (2010) are described to show how mathematics education can be improved for both the preservice teacher and their elementary students through the application of growth mindset and what Boaler calls a mathematical mindset. Their work compliments each other by providing the theoretical framework of growth mindset, the classroom practices that encourage a mathematical mindset, and a great visual for helping learners see how to maintain a healthy mindset in all subject areas.

Dweck has been studying the power of people's beliefs on their lives. She started to identify qualities in successful people of all ages, and she wanted to know how people coped with failure. She coined the phrases fixed and growth mindset to describe two main categories of beliefs that affect how people act and react to their environment. Dweck gave fixed and growth mindsets examples in people of all ages, from all professions including education. In fact, Dweck's research is applicable to preservice teachers learning how to teach mathematics effectively. Dweck (2006) wrote, "Because they think in terms of learning, people with the growth mindset are clued in to all the different ways to create learning" (p. 62).

Students with growth mindsets have characteristics that can help them be successful learners. They embrace challenges and are unafraid of the effort they must put in to "succeed even in the face of setbacks" (Dweck, 2006, p. 245). They learn from failure, from criticism, and even from the success of others.

A five-year study of 373 seventh grade students in New York City included an eight-week intervention program for low scoring mathematics students (Blackwell et al., 2007). Each student completed a questionnaire about their views on ability, effort, and intelligence to determine their initial mindsets. Their sixth-grade mathematics achievement test score and mathematics grades from the end of year seventh- and eighth-grade mathematics classes were obtained. This study's results showed no correlation between incoming seventh graders' theory of intelligence, or mindset, and their sixth-grade achievement test score. However, "as students made the transition to junior high school, their theory of intelligence became a significant predictor of their mathematics achievement" (p. 251). The students whose theories of intelligence leaned toward the growth mindset earned higher mathematics grades by the end of their eighth-grade year.

The second part of this 2007 study involved an eight-week intervention with 91 students who had all scored at or below the 35 percentile on their sixth-grade mathematics achievement test. These students were split into two groups: 48 in the experimental group and 43 in the control group. The experimental group participated in a 25-minute advisory lesson once a week during their seventh-grade spring semester. While all 91 students had lesson which included "instruction in the physiology of the brain, study skills, and antistereotypic thinking," (p. 254) the experimental group also completed lessons about their ability to develop and change their intelligence. Participants were surveyed again about their beliefs on intelligence, teachers were asked to name and describe students whom they felt had made positive changes over the semester, and mathematics grades were tracked. The research found that students who had more of a fixed mindset benefited the most from the experimental intervention. Many changed the trajectory of their grades while those students with fixed mindsets in the control group had "math grades that continued to decline" (p. 258).

Ricci (2015) conducted her own research studies to show how mindsets of children are affected even in the first years of elementary school. She interviewed kindergarteners and found that all of them had growth mindsets at the start of the school year. They agreed with statements such as, "I can learn anything I want." She then interviewed first-, second-, and third-grade students. She found 10% of first graders, 18% of second graders and 42% of third graders already exhibited traits of fixed mindsets. For example, during interviews students reported that, "Some people are smart, some people are not" (p. 11).

Students with fixed mindsets can become teachers with fixed mindsets. Boaler (2016) noted, "Many elementary teachers feel anxious about mathematics, usually because they themselves have been given fixed and stereotypical message about the subject and their potential" (p. 105). Teachers, both preservice and inservice, may see mathematics as a list of individual skills to teach if they study grade level standards or textbooks in their mathematics methods course curriculums without learning to emphasize how the concepts and skills are related to each other. It is hard to develop number sense and conceptual understanding, a mathematical mindset, when mathematics is seen only as a set of computational skills. This may be particularly true in the elementary grades where so much of the number and operations standards include place value and computation of whole numbers, decimals, and fractions. Boaler suggested that if instruction includes "real mathematics, a subject of depth and connections - the opportunities for a growth mindset increase, the opportunities for learning increase, and classrooms become filled with happy, excited, and engaged students" (p. 32). Boaler's comment seemed to help connect mindset research with the teaching of number and operations concepts which was the focus of this researcher's study.

Boaler described a mathematical mindset as "an active approach to mathematical knowledge, in which students see their role as understanding and sense making" (p. 36). Conceptual understanding of mathematics should be the goal of methods courses instruction to improve preservice teachers' content knowledge of mathematics resulting in lessons that build students' conceptual understanding. When teachers and students learn to "shed the harmful ideas that math is about speed and memory," (p. 55) then they can change their mindsets about mathematics and see that everyone can be successful in mathematics. Boaler also encouraged teachers and students to eliminate the notion that there is one way to reach a solution, and to "think about different methods, pathways, and representations" (p. 77) to solve problems.

Boaler's book, *Mathematical Mindsets: Unleashing Students' Potential Through Creative Math, Inspiring Messages, and Innovative Teaching* also included research and advice about making mistakes. In growth mindset classrooms, mistakes are seen as positive, learning experiences. She summarized research that showed brain activity actually increased when mistakes were made. Boaler suggested the teaching practice of allowing students to redo assignments and tests for better grades as a way to promote a growth mindset. Individuals with a growth mindset were more likely to go back and correct errors (p. 12).

Students with fixed mindsets and anxiety may display poor attitudes towards learning have a hard time learning, potentially perpetuating the negative cycle of beliefs. Students must see the power of their attitudes and understand how to improve them in order to be successful when faced with new challenges. Nottingham (2010) provided a visual model to help students with this. Nottingham first created the "Learning Pit" in 1999 working as a teacher in the United Kingdom. He told his students it was good for them to do challenging work, and described growth as a journey through a pit which has four stages - concept, conflict, construct, and consider. A task or concept is presented and students experience different levels of conflict (mild discomfort through extreme frustration). Students then try different strategies or solutions as they work their way out of the pit. In the end, students reflect on what they've learned and how they've grown.

2.1.3 Critiques and Concerns

Critiques, concerns, and new research studies about implementing growth mindset in education have arisen over the past years leading Dweck (2015) to publish articles clarifying mindset research and beliefs and correcting misconceptions and misapplications of her work. These concerns, new research from Kathy Liu Sun, and Dweck's clarifications are shared here.

Thomas posted numerous blogs in 2018 criticizing growth mindset. One concern was that educators seemed to believe that having a growth mindset causes success, while those that don't have it can't be successful. He stated that this oversimplifies a much larger problem. In a May 30, 2018 blog he wrote, "Teaching and learning as well as success and failure are incredibly complex", and he warned that educators shouldn't assume there is an easy fix for students and our education system. Thomas instead encouraged the reading of Gorski who promoted equity practices to reduce the opportunity gap between students in and out of poverty.

Kohn and Blad also posted online articles raising concerns about growth mindset which they believe is a deficit ideology, a theory that lays the cause of student failures on the students' own shortcomings. Both authors expressed concern that the focus on students' growth mindset, or lack thereof, would take the focus off improvements needed in the educational system itself. Blad believed that teaching growth mindset to struggling students comes from good intentions, but that educational systems should first address and fix the poverty, food insecurity, and transiency affecting students. Blad wrote in an August 8, 2018 article that if these issues were alleviated, educators would be "likely to find that those students in poverty who appeared to lack 'grit' and growth mindset would then demonstrate those treasured qualities."

Kohn's article included concerns about growth mindset being applied to education. He worried that educators who see mindset as the key factor to student success will spend less time questioning curriculum quality, teaching practices, and the educational system itself. In an August 16, 2015 online article, Kohn called the application of mindset to education a "fix-the-kid, ignore-the-structure mentality," and reminded readers that "no mindset is a magic elixir that can dissolve the toxicity of structural arrangements."

Kohn noted his concerns about what he sees as overpraising effort. He stated that it can "communicate that they're really not very capable and therefore unlikely to succeed at future task. ('If you're complimenting me just for trying hard, I must really be a loser.')." He suggested feedback to students that communicates how they've done on a task without judgement or grade to build intrinsic instead of extrinsic motivation.

Dweck has voiced many of the above concerns regarding the misapplication and misunderstanding of her research on growth mindset. She cautioned teachers and parents that effort is not the only key to educational success. The goal is to learn, and students who are not learning need to be told that and given strategies to improve. She stated that praising effort should not become a substitute for learning. She encouraged praising effort to teach students to embrace and work through challenges without giving up when learning something new.

Dweck stated, “I also fear that the mindset work is sometimes used to justify why some students aren’t learning: ‘Oh, he has a fixed mindset.’ We used to blame the child’s environment or ability” (p. 22). This seemed to address the concerns of Thomas, Kohn, and Blad who worried about growth mindset as a deficit ideology. Dweck wrote that educators should be continually focused on trying to find new ways to help struggling students understand and succeed.

Dweck also addressed false growth mindsets. People with false growth mindsets are those who profess to have a growth mindset, but don’t align their practices toward that mindset. Sun’s (2015) research focused on how teachers influence their students’ mindsets in terms of their mathematics attitudes and abilities, how mindset messages were communicated to students, and how instructional practices varied among teachers of differing mindsets.

Sun surveyed 40 California middle school mathematics teachers and approximately 3,100 and 3,500 of their students were pre and post-surveyed at the end of the school year. Sun also chose four pairs of teachers with different mindsets teaching the same grade and course for case studies. This included recordings of lessons and the first days of school when Sun theorized most teachers explicitly shared their views on learning and expectations for the classroom. Multiple students from each case study classroom were interviewed at the start and end of the school year as well to determine their mindsets.

Sun found that the teachers’ self-reported mindsets at the start of the school year did not predict their students’ mindsets by the end of the school year. Instead, her data seemed to indicate that the teachers’ beliefs and practices were more likely to predict a growth mindset in their students by end of year. The research suggested that the teachers’ beliefs about the nature of mathematics, specifically those who believed that “math was more than procedures,” (p. 82) resulted in more students reporting a growth mindset at the end of the year. In addition, teachers who felt that mathematics was accessible to all students were more likely to have students reporting a growth mindset at the end of the year.

Sun identified the four areas of teacher practice that could communicate either a fixed or growth mindset to students as sorting, norm setting, mathematics tasks, and feedback and assessment. She then created charts to describe "a continuum of the enactment of each practice rooted in the empirical observation of the case study teaching" (p. 94). She called this continuum the mathematics teaching for mindset framework. For example, teachers who grouped students according to one dimension of mathematics ability, had different expectations for low versus high performing students, and publicly posted student performance were on the fixed mindset side of the continuum. Teachers who created groups based on multiple aspects of mathematics ability, expected all students to contribute to mathematics assignments, and posted a range of student work recognizing multiple aspects of student performance were on the growth mindset side of this continuum.

In the area of norm setting, Sun described four sub-areas where teachers can communicate their mindset to students. On the growth mindset side of the continuum, Sun described teachers who explicitly talk with their students about growth mindset and how it relates to learning mathematics. Growth mindset teaching practices also address the importance of the process of learning and doing mathematics and the risk-taking that students should feel comfortable with as they struggle with learning mathematics. Sun also described the growth mindset practice of addressing mistakes as positive learning experiences that result in learning.

Sun's continuum described how the creation and implementation of mathematics tasks can convey messages of mindset. Teachers send fixed mindset messages when they choose tasks that are procedural in nature with only one solution path, and when they do the majority of the mathematics work because this sends the fixed mindset message that they are the authority figure. In contrast, a growth mindset messages are sent by creating or choosing tasks for the students to complete in multiple ways, possibly with multiple solutions. Growth mindset classroom practices utilize class time for students to share task solutions and encourage students to find similarities and differences between strategies.

The final area where Sun believed teachers communicate their mindset is the area of feedback and assessment. Like Dweck, Sun described a growth mindset teacher as one who provides specific praise highlighting not only effort, but the process of learning. A teacher can communicate a growth mindset by offering opportunities for students to receive additional help and to master and resubmit work that is graded on more than just the correct final answer.

Surveys, interviews, and classroom observations showed that many teachers reported a growth mindset, and many of these teachers also used the phrase growth mindset with their students. However, the students were more likely to have a growth mindset at the end of the year if their teachers' classroom and instructional practices aligned to a growth mindset. Sun noted a disconnect between the teachers' beliefs and their practices, and suggested that the context of the teachers' classroom had great impact on how their beliefs translated into classroom practices. This area of her research is particularly relevant to this research study in that it suggests possible barriers teachers face in implementing practices that align to their own teacher preparation and personal philosophies of teaching.

For multiple teachers in her study, Sun noted that coherence to district policies had an impact on classroom practices. One district paired with a local university to "design curriculum, train teachers, and implement a new standards based grading system" (p. 178). These teachers, regardless of their reported mindset, fell on the growth mindset side of the continuum compared to teachers in a neighboring district who had implemented a new program "primarily focused on teacher-centered math instruction that was very similar to scripted lessons" (p. 179).

The research summarized in the first section of this literature review described the effect teacher attitudes, confidence, anxiety, efficacy, and mindset can have on their own mathematics ability and teaching practices. Research included here gave examples of how mathematics methods courses could positively affect these teacher characteristics which is important to improving mathematics instruction. While there are concerns about the appropriate way to implement mindset research into classrooms, the research does appear to support the statement that teachers' ability, beliefs, and practices can have a positive effect on their own students' attitudes, anxiety, and mindset about learning mathematics.

2.2 Constructivism

Althauser (2018) stated that there is an advantage of the constructivist framework. "Constructivist approach to teaching, which encourages a deep learning approach on the behalf of the learner, promotes self-efficacy" (p. 56). Research summarized in the previous section explained that lowering math anxiety can increase self-efficacy. An advantage of a constructivist approach is that the self-efficacy of both teacher and student can be increased. In this section the history and beliefs of constructivism are addressed. Cochran-Smith et al. (2015) stated that "preparing students for future knowledge work requires new ways of teaching that are grounded in constructivist views of learning" (p. 109).

The Association of Mathematics Teacher Educators (2017) standards encouraged the building of classroom communities where students "construct meaning together" (p. 13) because mathematical understanding is developed when students "construct each level of thinking and reasoning" (p. 55). Research studies will be shared to build a case for applying constructivism in mathematics methods courses and elementary classrooms as a means of increasing both mathematical content knowledge, knowledge of students, and pedagogical knowledge for teaching mathematics. In most of these studies, participants also reported reduced math anxiety and improved confidence as byproducts of the studies' methodologies.

2.2.1 History and Beliefs

The constructivist theory has emerged and gained acceptance in education over the centuries. Socrates, Plato and Aristotle are early constructivists (Hursen & Soykara, 2012). Von Glasersfeld (1988) reported that constructivist theories of learning can be seen in early works of Giambattista Vico in 1710, Jean Piaget in the 1800s, John Dewey (1916), and Lev Vygotsky (1962). Each of the theorists added important clarification and application of constructivism. Hursen and Soykara (2012), for example, distinguish Piaget's approach as cognitive constructivism and Vygotsky's approach as sociocultural constructivism.

Piaget identified four stages of cognitive development and the general ages associated with each stage. Preservice teachers must learn that the sensorimotor stage (birth to two years), preoperational stage (ages two to seven), and concrete operational stage (ages seven to eleven) apply to elementary age students. Knowledge of different cognitive levels and strategies to be used at each level help teachers plan appropriate, differentiated, and effective lessons and tasks.

Dewey encouraged the use of manipulatives to introduce and learn concepts. He wrote that manipulatives "give the pupils something to do, not something to learn; and the doing is of such a nature as to demand thinking, or the intentional noting of connections; learning naturally results" (p. 160). Bruner's concrete, pictorial, and abstract (CPA) instructional method directly relates to Dewey's constructivist theory. Bruner believed that student understanding and learning progressed through these cognitive stages. Constructivist teachers use this information to determine the level of understanding of their students and find appropriate models or materials to use in lessons. However, this skill improves with practice and experience. Ortiz's (2017) study of 88 elementary preservice teachers found that only 51% could correctly select the correct CPA level of a given lesson plan's materials or manipulatives.

The central tenet of constructivism is that students build knowledge through doing and experiencing what is to be learned so they are active participants in their own learning (Althauser, 2018). New knowledge, gained through active participation, is connected to old knowledge making the scaffolding choices teachers make an important part of the teacher's pedagogical skill set (Shirvani, 2009). Therefore, the primary role of the constructivist teacher is to plan for and facilitate experiences that will make learning possible and meaningful for all students. Shirvani (2009) noted characteristics describing a constructivist classroom include personal relevance of mathematics, open and critical student communication, shared control between teacher and students, and positive attitudes towards learning.

The National Council of Teacher of Mathematics' (1989) *Curriculum and Evaluation Standards for School Mathematics* called for reform in mathematics education. Therefore, many teacher preparation programs have adopted constructivist philosophies in their mathematics methods courses. The following sections of the literature review focus on research studies that describe promising practices in constructivist mathematics methods courses whose curriculum and experiences increased preservice teachers' mathematics content knowledge, knowledge of students as they learn mathematics, and pedagogical knowledge. The authors of the research studies described here explain different ways in which they helped preservice teachers decompose "each math concept into developmental steps following a Piagetian theory of knowledge" (Dornoo, 2015, p. 83).

2.2.2 Applications for Building Mathematical Content Knowledge

Multiple researchers advocate for the use of manipulatives and pictorial models to help students and preservice teachers construct their understanding of whole number concepts and operations. Fuson and Briars (1990) used base-ten blocks, Hopkins and Cady (2007) used multilink cubes, and both Fernandez and Estrella (2011) and Roy (2014) used number lines and place value mats. Althauser (2018) created a self-efficacy survey with a list of preservice teacher experiences such as teaching word problems, answering students' questions about mathematics, helping students learn how mathematics is relevant to everyday life, and designing number and operations lessons which include hands-on lessons with a variety of manipulatives and/or technology. Valli et al. (2001) used the terms adaptability and flexibility to describe a teacher's ability to model different learning strategies and concrete manipulatives appropriately for different situations and students. Flexible teachers can positively and effectively react to whatever direction a lesson takes and "provide for alternative activities if something goes wrong" (Johnston, 2001, p. 9).

Quinn (1997) conducted a study with 47 preservice teachers (26 elementary preservice teachers) enrolled in a mathematics methods course. The Aiken's Revised Mathematics Attitude Scale and the Essential Elements of Elementary School Mathematics Test were administered at the start and end of the course. This course was designed to emphasize the use of manipulatives, technology, and cooperative learning in the teaching of mathematics, and stressed conceptual knowledge over procedural knowledge. It is interesting to note that educational researchers have distinguished between the two types of knowledge since the mid-1970s when Skemp (1976) referred to instrument (procedural) and relational (conceptual) understanding. The scale score of 43.3 on a scale range of 0-80 puts the attitudes of the average preservice teacher as "neutral." The post-test average was 70% on the content knowledge test. The scores showed statistically significant growth, but the author noted that they left room for improvement.

Kajander (2010) had the same "minimally acceptable level" (p. 228) of increase in the final results of her three-year study of preservice elementary teachers. The study had two goals. The first was to examine the development of preservice teachers' knowledge and understanding of mathematics as they completed their teacher education program. The second was to utilize interviews with 22 of the participants to examine beliefs about the importance of developing conceptual understanding of elementary mathematics concepts.

Over 300 preservice teachers took part in the study as they completed a two-semester mathematics methods course. Kajander (2010) used a Perceptions of Mathematics Instrument of 20 questions that she designed, pilot tested, and revised over a yearlong period because she wanted a short assessment that could separate procedural and conceptual knowledge of the participants. Results showed that participants generally had stronger beliefs on the importance of conceptual learning. They also developed stronger conceptual knowledge of the mathematics concepts covered by the courses although the development was minimal.

According to Kajander, there was further work to be done in improving mathematics methods courses because the preservice teachers still struggled in their ability to "provide explanations, models, or other evidence of alternative methods that indicated a deep understanding of standard mathematical procedures" (p. 242). Preservice teacher discourse allowed mathematics methods course professors in this study to determine what they did or did not understand. Razfar (2012) also encouraged the use of discourse because it "afforded a more holistic view of human meaning-making" (p. 48). If we expect preservice teachers to carry mathematical writing and discourse practices into their own classrooms, an important part of mathematics methods courses would be the use of mathematical writing and discourse as ways of explaining and understanding concepts for the preservice teachers first.

A very interesting twist to Kajander's (2010) study came from a small subset of 20 participants who enrolled in a separate, non-credit, 20-hour course called "Mathematics for Teaching." The course, designed by Kajander herself, and taught by an elementary mathematics teacher, was taken during the same semester as the mathematics methods course. All of these students passed Kanjander's course assessment with a score of 60% or higher which seemed to indicate that the additional course improved participants' procedural and conceptual understanding of the mathematics. This optional course, offered in the third year of the study, focused on the same mathematics content covered in the methods course, but many participants reported that it was beneficial for them to have additional time on the topics.

Kajander was not the only one to find that additional time improved content knowledge. Burton, Daane, and Gleisen (2008) designed and taught two methods courses simultaneously. The concepts taught in both classes were the same. However, one course was a traditional methods course, and the other course replaced 20 minutes of methods content with mathematical content. The Content Knowledge for Teaching Mathematics Measure developed by Hill, Schilling, and Ball in 2004 was used as pre- and post-tests. The researchers found that the additional 20 minutes of mathematics content knowledge lessons based at fifth and sixth grade levels resulted in greater growth in the participants' mathematics knowledge for teaching. This was despite the fact that the participants in the traditional methods course had a higher mean score on the pretest.

The 20 students in the experimental group spent the 20 minutes of intervention time on such topics as analyzing different representations of data, equations for area, perimeter, and volume, creating alternative notations for numbers, and examining standard and alternative algorithms for the four operations. The experimental group had a statistically significant increase in their mean score, but the participants without the intervention did not experience a statistically significant change in their mean scores (Burton, Daane, & Gleisen, 2008).

Ford and Strawhecker (2011) piloted a study that also showed positive feedback from a blended mathematics methods course combining content and pedagogy. The authors surveyed a cohort of early childhood preservice teachers at the beginning and end of a semester long mathematics methods course co-taught by a mathematics and mathematics education instructor. They believed "when preservice teachers take content and mathematics courses concurrently, the connections between mathematical topics have potential to become clearer" (p. 3). Three themes emerged from their qualitative data, the connection between content and methods, having empathy for those learning new material, and increased confidence in learning content even "beyond their desired teaching level" (p. 8).

Fast and Hankes (2010), mathematics methods instructors, conducted a study to determine whether intentional integration of mathematics content instruction could improve preservice teachers' understanding of constructivist pedagogy. The instructors had noticed over their years of teaching the course that many preservice teachers couldn't appreciate the pedagogies because they didn't have conceptual understanding of the elementary mathematics themselves. Many of the preservice teachers showed negative attitudes and frustration towards the mathematics which disrupted the lesson and kept them from “accomplishing the primary objectives of the lesson” (p. 331), showing the benefit of a particular constructivist pedagogy.

Fast and Hankes (2010) independently taught two concurrent mathematics methods courses, one as an experimental group and one as a control group. Each group took the same pre- and post-test assessing content knowledge and confidence in mathematics knowledge. The control group of 35 preservice teacher participated in a traditional mathematics methods course where constructivist pedagogies were introduced and modeled. The experimental group had the same pedagogies introduced and modeled, but the instructor specifically included mathematics content instruction before or during the activities. The researchers were interested in determining the effect of content instruction on knowledge and confidence, but they also wanted to determine if the course itself had enough time available for the added content instruction.

The results of the post-test showed a significant increase in content knowledge and confidence for the experimental group, not the control group. The instructor of the experimental group also reported that instruction of mathematics content related to a specific pedagogy of that content allowed the instruction to “proceed at an increased rate” and he was able to “complete, in the regularly allotted time, the usual instruction in pedagogy” (p. 335). The instructor’s theory was that the preservice teachers were able to “proceed at a much quicker pace in pedagogical instruction when the students already understood the content in the mathematics examples used to anchor the pedagogy” (p. 337). An additional benefit to the integrated content instruction was a decrease in students’ frustrations and math anxiety throughout the semester long course.

In addition to blending mathematics content and pedagogy, mathematics methods courses may introduce different kinds of manipulatives used to teach number and operations concepts to promote active engagement (Bamberger et al., 2010; Saran & Gujarati, 2013). Preservice teachers could then determine which manipulatives and “when and how math concrete models would be used to support learning” (Unlu, 2018, p. 68). This may prevent misconceptions about manipulative use. For example, some teachers may assume that students had enough exposure to manipulatives in first and second grade and don’t need them any longer. Some teachers may think the students can already reason abstractly about the new concepts being introduced.

Research didn't seem to support these opinions. In fact, Bamberger et al. (2010) specifically explain that students should spend a good deal of time with manipulative models before utilizing any new procedures with pencil and paper solutions. Upper elementary teachers are teaching new computational procedures and concepts that require the use of manipulatives to follow the same CPA instructional method used in primary grades. The authors' advice seemed to suggest that upper grade teachers would have fewer complaints about students' inability to subtract, multiply, and divide if they would follow the CPA framework. Many of the computational errors made by students "reveal an emphasis on digits and procedures rather than values and number sense" (Bamberger et al., 2010, p. 46). Building knowledge of how students learn mathematics and assessing what students understand is an important component of mathematics methods courses discussed further in the next section.

2.2.3 Applications for Building Knowledge of Students

Strawhecker (2005) stated that the "overlying goal of a methods course is to understand how children learn various mathematical concepts and skills and how to teach particular mathematical ideas to children" (p. 2). Mathematics methods courses can be designed to specifically facilitate the learning of preservice teachers as they construct their knowledge of elementary students. Preservice teachers are learning how students learn developmentally, and they need insight into how elementary aged students think about mathematics (Kastberg et al., 2013). Althauser (2018) stated that constructivist mathematics teachers must learn to "listen to learners in ways that allow them to build a model of each learner's mathematical knowledge" (p. 57). Unless preservice teachers have personal experience with teaching younger siblings or tutoring other children, "most of the teachers-in-training do not have an idea of what students in the elementary school know or understand" (Beal, 2001, p. 3).

Goodson-Espy et al. (2014) wrote that preservice teachers need experience analyzing student thinking and student work samples, to "assess student understanding, and in scoring student work with various rubrics" (p. 394). One activity for preparing elementary teachers which can be applied to their own classrooms is Pace and Ortiz's (2016) "Get the Goof!" lesson. Michelle Pace is a second-grade teacher who regularly displays student work for her students to analyze. They study the work to look for mistakes or simply to evaluate a new algorithm for adding or subtracting. Hill and Ball (2009) would appreciate the way this activity builds preservice teachers' "capacity to see the content from another's perspective" (p. 69). This activity and student interviews are effective strategies for giving preservice teachers opportunities to construct their knowledge of children's mathematical thinking.

A professional development program called Cognitively Guided Instruction (CGI) was developed by Carpenter, Fennema, Peterson, Chiang, & Loef (1989) which may be applicable to methods curriculum. The program was designed to increase teachers' understanding of what students understand about mathematics concepts and how to connect that understanding to formal concepts and operations. Moscardini's (2014) study showed that teachers who implemented CGI in their classrooms reported they were better able to notice students' mathematical strategies through formative assessments, discussions, and observations, and use those strategies to guide instruction and remediation. Carpenter et al. (1989) and Villasenor and Kepner (1993) found that students in CGI classrooms scored significantly higher on fact based and word problem solving tests.

It should be said that the ability to "understand what another person is doing entails mathematical reasoning and skill that are not needed for research mathematics" (Hill & Ball, 2009, p. 69). However, these skills are essential to effective teaching of mathematics. This is an example of how unique mathematical knowledge for teaching is, and it illustrates how connected mathematics content knowledge, knowledge of students, and pedagogical knowledge are when teaching mathematics for conceptual understanding. Baumert et al. (2010) writes:

> One of the major findings of qualitative studies on mathematics instruction is that the repertoire of teaching strategies and the pool of alternative mathematical representations and explanations available to teachers in the classroom are largely dependent on the breadth and depth of their conceptual understanding of the subject. (p. 138)

The collection and use of these strategies, representations, and explanations are components of building pedagogical knowledge for teaching mathematics. Althauser (2018) gave two suggestions about mathematics teacher preparation coursework. First, the curriculum should include constructivist pedagogical strategies that "require preservice teachers to manipulate materials and ideas in order to explore concepts and make connections between ideas" (p. 57). Second, the author stated that mathematics methods curriculum should include activities and lessons for preservice teachers that model the instructional strategies they are expected to use in their own classrooms.

The next sections highlights ways in which constructivist mathematics methods courses can have a positive impact on pedagogical knowledge for teaching number and operations by modeling these strategies. The specific domain of mathematical knowledge known as Mathematical Knowledge for Teaching (MKT) is discussed as well to build a case for including MKT instruction in mathematics methods coursework.

2.2.4 Applications for Building Pedagogical Knowledge

What is pedagogical knowledge? Why and how is it different from mathematical pedagogical knowledge? These are important questions for instructors to answer as they design mathematics methods course curriculum and experiences for preservice teachers. Halagao et al. (2009) explained that "Pedagogies represent the relationships among the purpose of education, the context in which education occurs, and the content and method of what is being taught and learned" (p. 4). In practical terms, general pedagogies include such things as "knowledge of instructional planning, student assessment, classroom management, how to facilitate group work, and how to address heterogeneity among students" (Youngs & Qian, 2014, p. 249).

Mathematical pedagogical knowledge is all of this and more. Pedagogical knowledge specific to the teaching of mathematics is the combination of "what one knows about mathematics, about students, about general pedagogy, and about learning mathematics" (Strawhecker, 2005, p. 2). Caughlan et al. (2017) pointed out the importance of methods courses as the place where "novice teachers encounter the specific pedagogical problems in a discipline and the specific instructional practices for addressing them as they intersect with the content that needs to be taught" (p. 270). The research studies included in this section give examples of promising practices for increasing mathematical pedagogical knowledge which can be introduced, modeled, and practiced in mathematics methods courses. The constructivist pedagogies included here are culturally sustaining pedagogies, inquiry-based lessons, mathematical modeling tasks, manipulatives, and games. Use of these pedagogies to differentiate instruction is also discussed.

Dewey (1916) wrote, “No one has ever explained why children are so full of questions outside of the school (so that they pester grown-up persons if they get any encouragement), and the conspicuous absence of display of curiosity about the subject matter of school lessons” (p. 162). Students are not interested in mathematics or other subject matter that doesn’t seem to pertain to them. Swars (2005) and McCoy (2008) found that providing authentic mathematics activities was an important teaching strategy for motivating students to learn mathematics. Authentic mathematics comes from the everyday lives of the students. This can be achieved when teachers gain knowledge and respect for the traditions, language, and culture of the community. Rosa et al. (2016) argued that this shows students especially of “underrepresented cultures that their own cultures do contribute to mathematical thinking” (p. 17).

Presmeg (1998) stated that the “ethnicity of students is a resource for mathematics teachers at all levels” (p. 318). She recommended that teachers integrate the ethnic and home activities of students into mathematics lessons to show the mathematics involved in the activities. Dominguez (2010) suggested redesigning mathematics curriculum to include out-of-school settings and experiences as a way to scaffold mathematics concepts especially for bilingual learners. Dornoo (2015) suggested that teachers ask themselves how their students’ “personal history and cultural context” (p. 85) can be used to explain mathematical concepts. This constructivist approach allows the students to start with what they already know and build new understanding of mathematical language and concepts.

Incorporating culture into mathematics is a critical piece of ethnomathematics which is, in turn, an important aspect of culturally sustaining pedagogy. Students can be shown that what they do and experience in everyday life is mathematics. “Beyond academic mathematics there lies a wealth of human activity that should be acknowledged as mathematical” (Mukhopadhyay et al., 2009, p. 75). Linking academic, school mathematics to students’ everyday experiences makes the mathematics “more relevant and meaningful to students” (Rosa & Orey, 2013, p. 76). These authors noted that integrating their students’ cultures within the mathematics curriculum allowed teachers to have more freedom and creativity in choosing topics and activities. Ma (1999) said it best when she wrote, “To empower students with mathematical thinking, teachers should be empowered first” (p. 105).

Two studies looked specifically to prepare preservice teachers to teach mathematics in a more culturally responsive manner to connect with their students. Aguirre, del Rosario Zavala, and Katanyoutanant (2012) asked preservice teachers to analyze the mathematics activities and lesson plans developed in class utilizing a rubric. The authors created a rubric which assessed how well the preservice teachers addressed the four aspects of culturally responsive mathematics teaching (CRMT) - children's mathematical thinking, language, culture, and social justice in their lessons. They found the highest self-rated averages were in the areas of analyzing children's mathematical thinking. The lowest self-rated averages were in the area of relating mathematics to language, culture, and social justice. The researchers argued that self-assessment of lessons, and teaching preservice teachers to specifically incorporate culturally responsive lessons into their mathematics classes are necessary components of a methods course.

McKinney, Berry, and Jackson (2007) studied how preservice teachers were being prepared to teach in high-poverty schools. The authors interviewed methods course participants to gather information about their coordinated 15-hour practicum experience. They were questioned about the challenges they encountered in the high-poverty school assignments, the instructional practices they observed, and asked for suggestions to improve the methods course to better prepare teachers in these schools. The researchers found the participants believed that more urban field experience and more careful, strategic choosing of supervisory teachers would improve their preparation program.

Choosing different types of mathematics tasks and problems is one aspect of mathematical pedagogies that can be introduced in mathematics methods courses. The tasks and problems can include higher order thinking through problems solving, project-based learning, discovery or inquiry-based teaching. Smith and Stein (1998) stated that choosing a good task or problem that will result in high levels of engagement and cognitive thinking begins with considering the students' ages, grade level, prior knowledge and experience, and the intended goal of the lesson. Teachers also need to consider what level of thinking the task requires, anticipate plausible thinking trajectories, and make predictions of student difficulties either conceptually or procedurally (Charalambous, 2010).

Once the task is chosen and planned out, the teacher must ensure that the implementation does not reduce the intended cognitive level. Yeats et al. (2005) noted a common mistake teachers make with a clear warning:

> The tasks are mathematically rich, but if teachers provide too many clues or too much specific help early in the process, they can stifle the deep thinking that the tasks can elicit from students. The challenge for teachers is to facilitate students' communication about a task without directing the students toward a particular solution. (p. 4)

This is a common mistake of even veteran teachers (Charalambous, 2010), so it is essential that preservice teachers are introduced to implementation strategies and given opportunities to practice these strategies within the methods course or in coordinated practicum experiences. Valli et al. (2001) found that beginning teachers were more likely to help students make sense of mathematics, without giving away answers or procedures, if they felt well prepared to teach connections between mathematics ideas and to teach problem solving skills.

Althauser (2018) and Magee and Flessner (2012) wrote about the impact of inquiry-based teaching on preservice teachers and elementary students. Both studies gave practical advice to instructors of mathematics methods courses for introducing, practicing, and improving this teaching pedagogy. Althauser (2018) used the 5E Instructional format (engage, explore, explain, extend, and evaluate) to guide preservice teachers in the planning of tasks based on deconstructed standards to teach the preservice teachers "how to use standards to support their curriculum selections and choices for instructional activities" (p. 59). At the end of Althauser's study, 78% of the 347 participating elementary preservice teachers reported higher levels of teacher efficacy as a result of the methods courses and practicum experience.

Magee and Flessner's (2012) study gave additional advice for methods instructors and their preservice teachers by showing how mathematics and science content can be integrated in inquiry-based activities. Again, selecting the task was the first consideration. The authors suggested two criteria for choosing tasks: 1) the questions should be mathematically challenging, and 2) the questions should not be able to be answered by a quick internet search.

Magee and Flessner's (2012) offered suggestions of the steps for preparing preservice teachers to implement inquiry lessons. First, the preservice teachers participated in an adult-level inquiry project investigating something they found personally interesting. Second, the preservice teachers participated in an inquiry experience based on content they would teach in elementary mathematics. Finally, the preservice teachers developed, implemented and assessed their own inquiry-based lesson with elementary aged children. This is not only a powerful method for preparing teachers in inquiry, but a perfect example for preservice teachers in scaffolded learning of difficult tasks and concepts!

In the preservice teachers' journals and lesson reflections at the start of the methods course, Magee and Flessner (2012) found that many teachers were concerned with how unstructured and chaotic inquiry-based lessons seemed. By the end of the study, preservice teachers' journals included comments about how powerful the lessons were when students were allowed to "take control of their own math learning during our sessions" (p. 360). Another teacher noted that the student discussions during these lessons were "beneficial because it allows students to learn from one another and bounce back ideas from another" (p. 362).

Judith Fraivillig (2001) also provided advice in the area of problem solving which can be incorporated into mathematics methods coursework. She described a framework with three components - eliciting children's solution methods, supporting children's conceptual understanding, and extending children's mathematical thinking. The first component is a listening, encouraging, and clarifying stage for teachers as they learn what student do and do not understand about a problem or concept. Next, teachers offer support, not answers, while students work by reminding them of past problems or situations, giving background knowledge for the problem's context, and encouraging students to ask for specific assistance when needed. This is also the stage where teachers write symbolic representations of the solutions shared by students on the board for class discussion. In the last component, teachers discuss the various traditional and alternative solutions in terms of accuracy and efficiency. This is an important time for teachers to model enthusiasm and love of challenge in order to pass those traits on to the students. This same love of challenge, much like Dweck's growth mindset, must be evident in the mathematics methods course as well.

Problem solving is also a valuable assessment tool. Charlesworth and Leali (2010) stated that "A problem solving task can be used to assess students' knowledge as well as, observe the students' thinking or reasoning abilities" (p. 380). They also noted that the task should allow students to explain their thinking with words, pictures, or a combination of both. This is the "P" of the CPA instructional method from Bruner discussed earlier. The authors noted that problem solving assessments are valuable tools even in pre-kindergarten and kindergarten classrooms where "observation is a major assessment approach" (p. 375).

In 2016, the Consortium for Mathematics and Its Application (COMAP) and the Society for Industrial and Applied Mathematics (SIAM) co-published the Guidelines for Assessment & Instruction in Mathematical Modeling Education (GAIMME) to encourage the use of authentic, culturally relevant problems and tasks to teach mathematical concepts from elementary through secondary education. The modeling that GAIMME referred to is not the concrete or pictorial models discussed earlier, but a six part process much like the engineering design process associated with STEM lessons. In the first step, students identify something in the real world that they want to know, do, or understand. They must then determine what information they need, and what mathematics will be needed for the model. After analyzing their information and possible solution, the students make any necessary revisions and implement the solution. The diagram in Figure 1 is from the GAIMME report (p. 13).

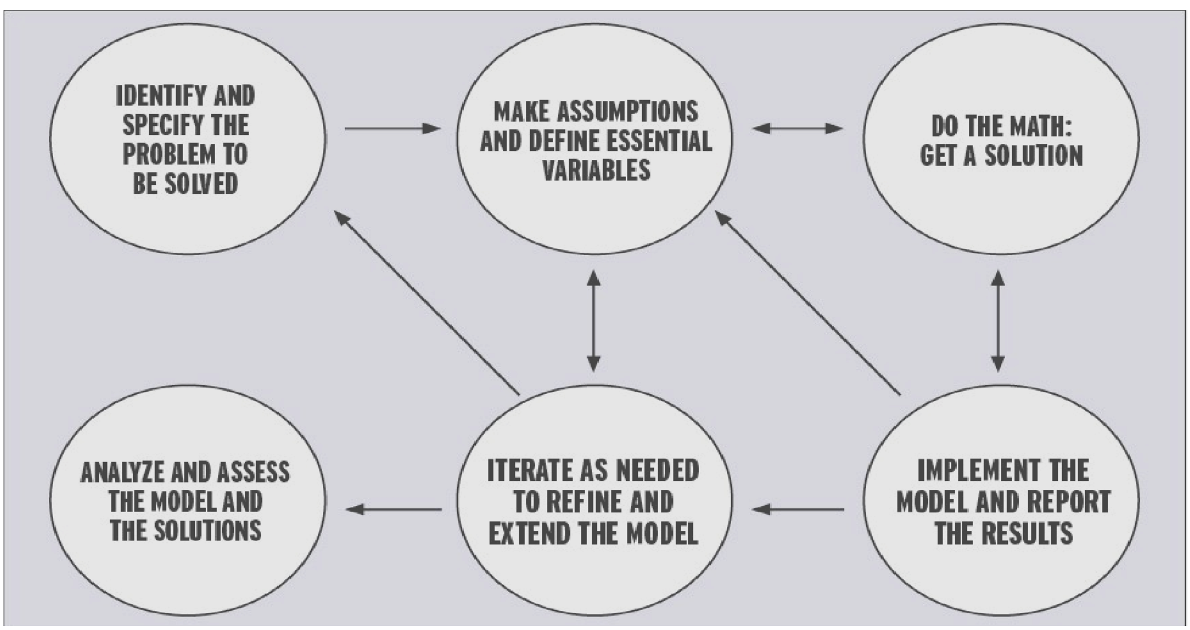

Figure 1. Mathematical Modeling Process. This figure shows the six-part process described in the Guidelines for Assessment & Instruction in Mathematical Modeling Education (2016).

Mathematical modeling tasks offer another avenue for students to build their mathematical knowledge and conceptual understanding of topics. These tasks are different from typical word problems or performance tasks in that the students must determine the information needed to solve the problem. Teachers must know what models students are working on to determine what mathematics procedures or concepts they will need to solve the problem. These problems can come from all content areas. The GAIMME report offered examples of mathematical modeling tasks, checklists, and assessment rubrics for different grade level bands. As an example, the GAIMME report suggested the “Lunch Box Problem” (p. 119). The question posed was, “What should go in a lunch?” Each student then determines how they will answer this question. Will it be based on calorie count, the study of volume, the costs of the lunch’s contents, or some other aspect of personal interest? The mathematics involved in each lunch box scenario is different and unique to the students. The GAIMME report offered suggestions for possible mathematics connections from prekindergarten through high school which can be used as a closing discussion.

Asking preservice teachers to complete their own mathematical modeling problems is an effective way to introduce them to teaching this type of task similar to Magee and Flessner’s (2012) introduction of inquiry-based lessons. By experiencing this type of lesson themselves, they can see the benefits of modeling and be better prepared to facilitate this type of lesson. The first-hand experience can help them anticipate management and content issues that might arise, and help them think of possible solutions ahead of time.

Differentiation strategies that ensure appropriateness, access, and success for all students can also be introduced, modeled, and practiced in mathematics methods courses and practicum experiences. Differentiation strategies are typically grouped into categories, differentiated content, differentiated process, or differentiated products. In past conversations with colleagues, differentiation was frequently named as an area of weakness. Mathematics methods courses can address this issue to prepare preservice teachers for the diversity they will likely encounter in elementary schools. The “teachers’ inability to teach in a way that is appropriate to the level of the developmental stage of the learners” (Peker, 2009, p. 342) may be a cause of math anxiety for both the teacher and his or her students.

In order to differentiate, teachers must know their content and their students well enough to design tasks and carry out the lesson's intended implementation. Ball (2000) wrote that "the creativity entailed in designing instruction in ways that are attentive to difference requires substantial proficiency with the material" (p. 242). Hill (2018) added that teachers must know how to "construct problems with similar interpretations" (p. 523). For example, how can a task for teaching perimeter be created and written for students with high, average, or low mathematics and/or reading ability? This is an especially important skill for differentiation because it ensures that all students are given access to the same mathematics content at their own level of ability or understanding. The teacher must then know how to sequence and scaffold the opportunities given so students can grow and be introduced to new concepts.

Problem solving tasks, such as those mentioned earlier, are appropriate to all students when the "complexity of solution methods are varied and they receive different degrees of scaffolding from the teacher" (Fraivillig, 2001, p. 458). Mathematics methods courses can work to increase the chances that preservice teachers will enter their own classrooms with the belief that all students deserve rich mathematical experiences, and the practical knowledge for how to create those experiences. One way to offer differentiated problem solving is through culturally responsive or culturally sustaining pedagogies where teachers create "tasks and approaches that are particularly important for different populations of students" (Ball & Forzani, 2009, p. 507). This approach provides students a way to make connections between academic mathematics and their everyday lives so they appreciate how useful mathematics can be. These lessons also highlight how all cultures contribute to and participate in mathematics equally.

Manipulative use, hands-on activities, and technology go hand in hand in effective mathematics classrooms and promising mathematics methods courses (Carbonneau et al., 2013; Fuson & Briars, 1990; Puchner et al., 2008; Unlu, 2018). These three pedagogies have the ability to be combined with other pedagogies to improve instruction (Carbonneau et al., 2013). For example, problem solving tasks can be hands-on or technology-based activities. Carbonneau and colleagues conducted a meta-analysis of 55 studies with kindergarten to college age students to compare instruction with and without manipulative use. These studies' participants ranged from kindergarten to college age students. Findings suggested that manipulative use had a small to medium effect on student learning compared to lessons which used “abstract symbols alone” (p. 396). Their finding that manipulative use, in conjunction with other pedagogies, increased the effects of the manipulative use is consistent with the research of Puchner et al. (2008) and Unlu (2018). Manches et al. (2009) and Quinn (1998) found that technology could provide effective virtual manipulatives for use as well.

Manipulative use can be used to differentiate instruction, providing concrete models for students struggling with the conceptual understanding needed for a task. A specific example of differentiated manipulative use can be found in the study conducted by Fuson and Briars (1990). The authors worked with 169 first and second graders in Chicago learning multidigit addition and subtraction using base-ten blocks. These students were homogeneously grouped into high, medium, and low ability mathematics groups by their teachers, then offered instruction in the use of base-ten blocks to model and solve problems. On the pretest, only nine of the students showed a knowledge of trading to correctly solve problems. However, on the post-test 160 of 169 students showed evidence of correct trading and problem solutions. The teachers reported that children were “enthusiastic about the multidigit instruction and enjoyed solving large problems” (p. 192). In a similar study, Robinson and Adkins (2002) noted that the mathematics methods course preservice teachers’ “anxieties could have in fact been prevented in elementary school, if they had received instruction through concrete manipulatives” (p. 6).

Using games and centers can also be an effective constructivist pedagogy for building understanding of mathematics concepts for both preservice teachers in mathematics methods courses and elementary students. The games may be concrete or virtual (Kermani, 2017). While they are frequently used as centers activities, games are also powerful in small (Vapumarican & Kapur, 2012) or whole group settings with some careful considerations.

McFeetors and Palfy (2017) and Olson (2007) discussed the importance of careful game choice. The concepts and skills, both mathematical and strategic, needed to play (and win) the game must be identified, and must match the intended goal of instruction. Once the game is chosen, the implementation requires just as much planning. There are many issues to consider in the effective use of games. How will the game be introduced - whole or small group? Will the game be an introductory, review, or enrichment activity? What differentiation may be required by students? Are there manipulatives that will help students? How can the game be made more challenging for the students who catch on quickly or win too often? What should student interaction sound like during the game? This list of questions seems intimidating, but answering them becomes habit with practice, successes, and failures just like everything in teaching. Mathematics methods coursework can provide the opportunity for preservice teachers to practice the use of these questions during the planning and implementation of game-based lessons.

Many researchers (Bittner & McCauley, 2015; Cavanagh, 2008; Heshmati et al., 2018; Pilten et al., 2017; Vandercruysse et al., 2017) reported positive results for preservice teachers' and students' learning of mathematics through games as well. However, there were implementation strategies that made the games more effective teaching tools. Vandercruysse et al. (2017), Heshmati et al. (2018), and Kermani (2017) noted that teacher questioning strategies and the integration of content impacted a game's effectiveness in teaching mathematics concepts. Games can have the mathematics content intrinsically or extrinsically integrated into the playing of the game. When games intrinsically integrate content, the mathematics skills and concepts are needed for the playing of the game itself. In extrinsically integrated games, students play games as a reward for completing a mathematics task. While most research cited by Vandercruysse et al. (2017) showed intrinsically integrated games as more effective for student gains in mathematics proficiency, the authors' own study found the extrinsically integrated game more effective for student gains in mathematics proficiency.

This section of the literature review included studies that showed how pedagogical knowledge and strategies can be adapted and applied to teaching mathematics. The next section will discuss the specific mathematical pedagogical knowledge base that preservice teachers develop through their teacher preparation coursework and years of experience. The authors included in the coming section call this knowledge base Mathematical Knowledge for Teaching.

2.2.4.1 Mathematical Knowledge for Teaching (MKT)

Preparation for teaching mathematics is a complex combination of necessary attitudes, confidence, and mathematical content knowledge. The Association of Mathematics Teacher Educators (2017) included standards addressing these aspects of teacher preparation. However, it is important to note here that most of the researchers, and AMTE, also refer to a specific domain of mathematical knowledge much like the optional course designed by Kajander (2010). This domain is best referred to as Mathematical Knowledge for Teaching, and it is connected to and frequently synonymous with pedagogical knowledge. In fact, when this domain was first introduced by Lee Shulman in 1986, he actually considered it pedagogical content knowledge, and called it the "missing paradigm" (p. 6) in teacher preparation.

Ball et al. (2008) helped create a concept map for the different domains of Mathematical Knowledge for Teaching (MKT). The following diagram in Figure 2 shows these domains and their relationships with one another (p. 403). Some content knowledge is the same in teaching as other professions, but other types of knowledge are specific to teaching and are best described as pedagogical content knowledge.

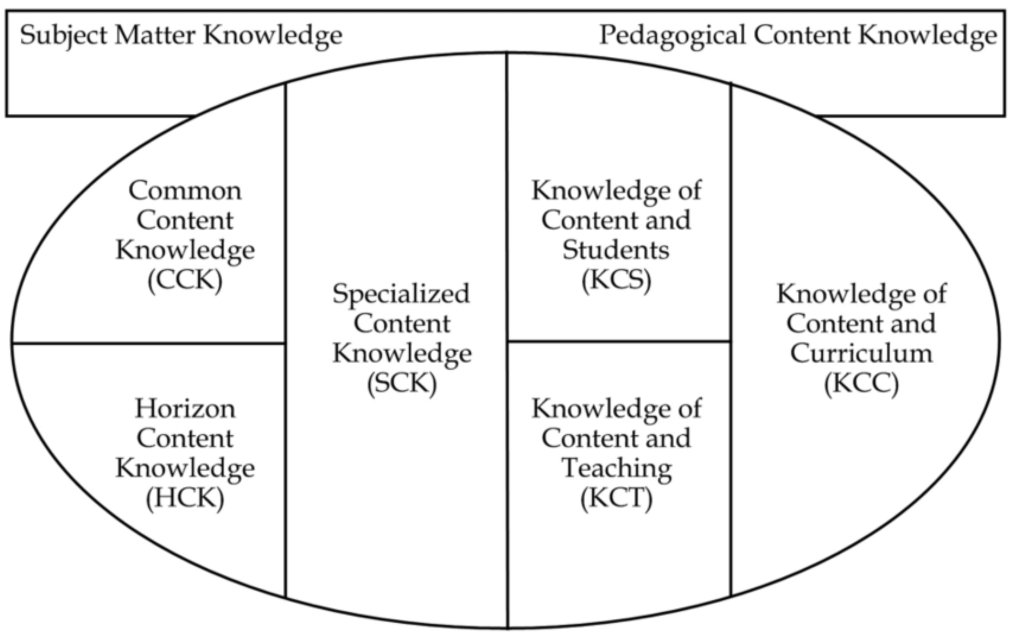

Figure 2. Domains of Mathematical Knowledge for Teaching (MKT). A concept map showing the different aspects of MKT created by Ball et al. (2008).

Mathematical Knowledge for Teaching (MKT) is a critical element in effective teaching. In fact, both Ball et al. (2005) and Charalambous (2010) found a relationship between teachers' MKT and their decisions and actions during the planning and implementation of mathematics lessons. This was true even for veteran teachers. Charalambous (2010) analyzed nine videotaped lessons from each of two veteran teachers chosen from a larger sample of teachers who had taken a paper/pencil Learning Mathematics for Teaching test. One veteran of 37 years scored in the 93rd percentile, the other veteran teacher of 23 years scored in the 35th percentile. The researcher identified the cognitive level of the curriculum lessons chosen by each teacher, revisions made by the teacher, and the final implementation of the lessons.

Through lesson plan analysis and analysis of the lessons' videos, he found that the teacher with higher MKT planned for and carried out lessons with higher cognitive demands, sometimes revising curriculum lessons to elevate the cognitive demands of the lessons. The majority of the lessons chosen by the teacher with lower MKT were of low levels of cognitive demand, and many times, she lowered the level further during the implementation of the lesson. For example, in a lesson on finding the area of a triangle she changed the inquiry lesson, giving the students a supplemental worksheet with the formula for area of a triangle. These findings in the one veteran classroom seemed to suggest that choosing and implementing tasks at high cognitive levels is challenging. In fact, the AMTE (2017) included a standard entitled, "Tools, tasks, and talk as essential pedagogies for meaningful mathematics" (p. 58) to promote the use of tools, tasks, and talk in teacher preparation program coursework.

Mathematics methods courses are designed to build preservice teachers' MKT so that students are in mathematics classrooms where authentic, engaging tasks are implemented consistently to build their understanding of mathematics concepts. A number of different teaching pedagogies and strategies that can be used independently or in combination to create this type of classroom environment have already been discussed. Modeling of and practice in utilizing these strategies is part of many mathematics methods courses.

Ensuring preservice teachers learn how to teach mathematics and how students learn mathematics through practicum experiences or simulated, micro-teaching practice is an application of the constructivist theory. Ball and Forzani (2009) stated that teacher preparation curriculum must become "practice-based" (p. 503) to give preservice teachers multiple opportunities carrying out the work of teaching, not just talking or theorizing about teaching. Research from Kastberg et al. (2013), Unlu (2018), and Basturk and Tastepe (2015), previously mentioned, offered videotaping and micro-teaching as options for teacher practice.

Ball (2000) stated that preservice teachers need help "finding ways to integrate knowledge and practice" (p. 244) in order to develop the resources and skills they need for effective teaching. Darling-Hammond (2010) wrote, “It is impossible to teach recruits how to teach powerfully by asking them to imagine what they have never seen or to suggest they ‘do the opposite’ of what they have observed in the classroom” (p. 42). Youngs and Qian (2013) found a positive relationship between preservice teachers’ opportunities to practice teaching and their Mathematical Knowledge for Teaching specifically in the area of number and operations. Student teachers who had full instructional responsibility during their field experience also scored significantly higher in Mathematical Knowledge for Teaching in number and operations.

Practicum experiences help preservice teachers build skills for managing materials, instructional pacing, and student behavior. Strawhecker (2005), Gokalp (2016), Aguirre et al. (2012), and Chigeza (2017) agreed that practicum experiences aligned to the theories and practices learned in methods courses offered preservice teachers a chance to apply what they’d learned about students, content, and pedagogy. Cochran-Smith et al. (2015) cited research encouraging preparation programs to increase “the amount of time teacher candidates spend in schools and beginning school-based experiences earlier” (p. 111).

If the quantity of field experiences doesn’t or can’t change, it is important to ensure a high quality of cooperating teachers and practicum placements. Darling-Hammond (2010) described research findings that suggest teacher preparation programs that keep “careful oversight of the quality of student teaching experiences” (p. 40) graduate teachers whose students show stronger achievement gains. Preservice teachers need an environment that, “involves learning from and with others by exchanging ideas, articulating reasons behind instructional decisions,” and “reflecting on one’s teaching to improve student learning” (Cochran-Smith et al., 2015, p. 111). Aguirre et al. (2012) and McKinney et al. (2007) stressed the importance of providing preservice teachers with opportunities to partner with supervising teachers who are successful in diverse classrooms to witness mathematical thinking from children with culturally and linguistically diverse backgrounds of all socio-economic levels.

Video-taping practice, micro-teaching lessons was another way to prepare preservice teachers, especially when practicum experiences are limited in quality or quantity (Basturk & Tastepe, 2015; Kastberg et al., 2013; Unlu, 2018). It allowed not only practice teaching, but analyzing and perfecting of pedagogy. The double benefit to viewing the videos was that preservice teachers learned not only through their own mistakes and successes, they also learned from watching the lessons of the other preservice teachers.

Mathematics pedagogical knowledge is a powerful component of effective teaching. It helps the teacher make connections to students, and it helps students connect to the mathematics content they are learning. Practicum and micro-teaching experiences are means by which preservice teachers can practice applying their mathematical content knowledge, knowledge of students' learning of mathematics, and pedagogical knowledge for teaching mathematics. Research shared in this paper point to successful mathematics methods models and frameworks that had a positive impact on the preservice teachers and elementary students. The next section of this literature review focuses on the effects of mathematics methods courses on beginning teachers' own classroom practices during their first years of inservice teaching. The extent to which first year teachers implemented the learning of their mathematics methods course(s) in their own classrooms and what barriers may exist to implementation is discussed.

2.4 Mathematics Methods Courses and Beginning Teacher Practices

The previous sections of this literature review highlighted studies that showed how mathematics methods coursework positively impacted preservice teachers during their teacher preparation program. Robinson and Adkins (2002) wrote "It is important that the beliefs strengthened in a mathematics methods carry forward to actually teaching experience, not only saying the right words, but demonstrating those beliefs and attitudes through behaviors" (p. 31). There are studies that concluded not all first year teachers implemented the philosophies and pedagogies of their teacher preparation program in their own classrooms. This section of the literature review includes summaries of first year teaching studies to try to understand which philosophies and pedagogies were more likely to transfer to first year teachers' classroom practices. These studies also provide insight into conditions that encourage or hinder the transfer of preparation program learning to first year teachers' classroom practices.

Research into first year teachers' practices and their connection to teacher preparation program teachings is limited. Sleeter's (2014) analysis of 196 education articles published in 2012 showed that only 6% examined the impact of teacher education on teaching practices. Sleeter also commented that many of the educational studies were lacking qualitative data which would provide details about the studies and their participants to help readers "visualize the applicability of findings to local contexts" (p. 151).

While there may have been fewer studies conducted in this area, several researchers are asking important questions about first year teachers' practices. In a 2010 essay, Diez questioned, "Are they doing what they learned?" (p. 444). It could be argued that this is the number one, most important question any teacher can ever ask of their students. It also applies to preservice teachers making the transition to independent, inservice teaching. Kastberg and colleagues (2013) called the long-term effects of the teacher preparation program "residue" (p. 1350). The attitudes and confidence, content knowledge, and pedagogical knowledge needed for effectively teaching mathematics, as they were prepared to do, may be considered examples of residue from mathematics methods coursework. Transferring and implementing philosophies and strategies from coursework into classroom practices is what Haggarty and Postlethwaite (2012) called "boundary crossing" (p. 246). These researchers warn that not all new teachers showed signs of residue or the ability to cross boundaries during their first year of teaching.

There are important questions that need to be asked next. What aspects of mathematics methods coursework are long lasting? What improves boundary crossing, and what barriers exist to implementing philosophies and strategies experienced in mathematics methods courses? Diez (2010) offered four reasons why first year teachers aren't doing what they learned in their teacher preparation program. Along with other researchers (Assen, Meijers, Otting, & Poell, 2016; Ensor, 2001; Hart, 2004; Jansen, Berk, & Meikle, 2017; Valli et al., 2001), Diez stated that some new teachers reverted to teaching styles and practices they experienced in their own education. Diez suggested teachers may revert to traditional teaching styles because they may lack confidence in their own content or pedagogical knowledge. In addition, teachers may lack confidence in their students' abilities and be hesitant or resistant to implementing what they learned in their preparation coursework. This lack of confidence could also make teachers give up quickly if faced with challenges while implementing strategies learned in their programs.

Implementing constructivist lessons that focus on conceptual understanding of mathematics topics was challenging for participants in studies by Valli et al. (2001) and Jansen et al. (2017). Both studies found that new teachers were more likely to teach for conceptual understanding if they felt "well" or "very well" prepared for teaching the content and for utilizing a particular teaching pedagogy. Jansen and her colleagues found that new teachers were "better able to enact instruction for conceptual understanding when teaching mathematics content that was developed in their elementary mathematics teacher education course work" (p. 245). Ensor (2001) found that teachers were able to "reproduce tasks that were introduced in the course," but they "could not produce new tasks that were analogous to them in terms of the approach to teaching that they privileged" (p. 314).

School or district mandated curriculum and pedagogy, resource challenges, and problematic work environments were the other three reasons Diez gave for why new teachers may not be implementing aspects of their teacher preparation program. These three areas are examples of the context for teaching that Sun mentioned in her dissertation when she theorized why teachers' practices didn't match their self-reported mindsets.

Multiple researchers (Chigeza et al., 2017; Cochran-Smith et al., 2015; Ensor, 2001; Haggarty & Postlethwait, 2012; Hart, 2004; Haynes, Maddock, & Goldrick, 2014) appear to support Diez's statement that mandated curriculum and pedagogy are barriers to the residue of teacher preparation coursework. Haynes et al. (2014) stated that new teachers in high-poverty schools are frequently told to follow mandates for what and how mathematics is taught with "extensive requirements for test preparation" (p. 7).

One participating teacher in the study by Chigeza et al. (2017) noted that his district mandated lessons on fluency with less focus on problem solving and reasoning. A teacher in Ensor's (2001) study stated she lacked autonomy, teaching from a pacing guide with preset lessons and tasks developed by veteran teachers that focused on "transmitting mathematical rules and procedures, breaking tasks into smaller steps, and frequently checking on learning" (p. 312). Prescribed course content and instructional pacing was also mentioned by Cochran-Smith et al. (2015) as barriers to new teachers trying to implement aspects of their preparation coursework.

There are examples of how new teachers are incorporating their preparation program's teachings into classroom practices in limited capacities. A teacher in Haggarty and Postlethwaite's (2012) study stated she was going to teach in ways she thought best "without obviously flouting school policy/contradicting those I am working with" (p. 257). The teachers in Hart's (2004) study maintained their beliefs developed in their mathematics methods coursework even after their first year of teaching. However, they all experienced "difficulty in working within traditional mathematics cultures" (p. 86). Many were hesitant to speak up or challenge their colleagues or administration, but they did attempt to include some tasks and behaviors learned in their preparation coursework into their classroom practices and lessons.

Resource challenges were mentioned by Diez and other researchers (Cady et al., 2006; Linek, Sampson, Haas, Sadler, Moore, & Nylan, 2012; Santoyo, 2016). Large class sizes, scheduling issues that result in lack of time for inquiry and hands-on lessons, and lack of supplies such as paper, texts, and technology are examples of possible resource challenges teachers face. These challenges may be especially discouraging to new teachers who may not have strategies for dealing with these setbacks. Teachers in the study by Linek et al. (2012) had concerns about time management and writing lesson plans. Teachers in the study by Cady et al. (2006) worried about having time to find, create, and implement tasks and discussions. Struggling with these challenges left the teachers in Spangler's (2013) study feeling "stress, uncertainty, and extreme fatigue" (p. 88). Being a first-year teacher may already put the novice teacher in a potentially challenging or frustrating position as they "experienced a unique combination of identifying as both students and teachers" (Santoyo, 2016, p. 26).

The final context that may be a barrier for new teachers trying to implement aspects of their preparation program is the school's culture itself. Diez called it a "problematic work environment" (p. 445). She noted that negativity and cynicism, even lack of professionalism among coworkers could discourage new teachers. Hart (2004) suggested that "novice teachers respond to the pressure of the school, the classroom, and other teachers and that many of their beliefs are based on these influences" (p. 80). Dangel's (2011) *Analysis of Research on Constructivist Teacher Education* included research that identified "supportive feedback, encouragement, openness to new ideas, and a positive climate as key factors that influenced the practice of a constructivist approach" (p. 15) to teaching mathematics.

In order to provide support to new teachers, district mentoring and induction programs are put in place. Valli et al. (2001) found that mentor frequency had a positive impact on effective mathematics instruction and student learning. New teachers that reported receiving monthly or weekly feedback on teaching, classroom management, and instructional planning had a stronger impact on student learning. Haynes et al. (2014) used the term "social capital" (p. 4) to describe the importance of interaction between teachers and administrators that were focused on student learning. The authors expressed concern for new teachers in schools serving high-need students because they may lack peers and mentors with the same preparation background and mindset. Teachers in Spangler's (2013) study ranked consulting with other teachers as the second source for learning about teaching after direct experience in the classroom.

Santoyo (2016) noted that the teachers in her study had support from "like-minded university and school-based mentors," but when they entered their first-year teaching assignment "they lost this cohesive environment" (p. 139). The teachers in this study were more likely to continue implementing constructivist lessons similar to those experienced in their preparation program if they had mentors who also implemented and valued constructivist lessons. This seems to suggest that carefully chosen mentors are a way to encourage transfer of preparation coursework into classroom practices, a means to help new teachers cross boundaries.

The first year of teaching is an important, challenging year for new teachers which can either reinforce or diminish the preparation they received. MetLife is a corporation that has sponsored yearly surveys of American teachers from 1984 through 2012. Harris (1991), who conducted the survey for MetLife, interviewed 1,007 public school teachers in New York before and after their first year of teaching. The first survey asked teachers about their expectations for their first year and their beliefs about teaching such as whether they felt they could "really make a difference in the lives of their students" (p. 3). After one year of teaching, the results showed a negative impact in this area. Teachers were also asked their perceptions on how well their training had prepared them to work with students from diverse backgrounds. After a year of teaching, there was no change in the percentage of new teachers that felt they would have benefited from more "practical training" (p. 12).

Other researchers suggest the need for more studies of first year teachers. Sleeter (2014) suggested that education departments in universities and colleges follow their graduates into their first years of teaching. Diez (2010) and Darling-Hammond (2016) suggested that new teachers build portfolios of lessons and reflections during their first years of teaching. Darling-Hammond (2016) favorably reviewed performance assessments used with beginning teacher in Connecticut and California. She compared them to National Board assessments in that new teachers "document their plans and teaching for a unit of instruction, videotape and analyze lessons, and collect and evaluate evidence of student learning" (p. 88). These activities and the feedback received may be a way to ensure residue from preparation programs so that the attitudes, content knowledge, and pedagogical knowledge learned will transfer into the teachers' practices.

2.5 Summary

Research suggests that "Effective mathematics teaching requires teachers' positive beliefs about mathematics, content knowledge, and knowledge of how to teach mathematics, and sociomathematics authority" (Saran & Gujarati, 2013, p. 101). This research can be used to guide teacher preparation programs in the creation of mathematics methods courses that will "prepare teachers as well as possible before they enter their own classrooms" (Kajander, 2010, p. 246). The experiences in methods courses designed to prepare candidates to teach number and operations include planning lessons, analyzing and revising textbook tasks, presenting and explaining terms, concepts, and procedures, using representations accurately, providing real world examples and applications of mathematics concepts, and interpreting and evaluating student work and errors (Charalambous, 2010). Providing practicum placements in conjunction with methods courses is an ideal way to offer firsthand experience to experiment with new pedagogical knowledge (Strawhecker, 2005).

Teacher preparation programs can benefit from preservice and beginning teachers' feedback about methods courses (Kastberg et al., 2013). Participants of Bekdemir's (2010) study suggested that teacher candidates should know their own math anxiety level, work towards a productive mathematics disposition, and "be warned not to transfer their anxiety to their prospective students" (p. 325). Participants in Chigeza et al. (2017) wrote about the disconnect between beliefs and practices shared in methods courses and those experienced in practicums. Many colleges may be eager, almost desperate for cooperating teachers for practicums, but it may be more important that cooperating teachers are specifically chosen. They should believe in and model the same qualities that mathematics methods courses are trying to encourage. If preservice and beginning teachers can't find "like-minded and knowledgeable colleagues for support" (Kajander, 2010, p. 246) they will likely revert to the "traditional teaching beliefs" (Cady et al., 2006, p. 302) of their own elementary mathematics instruction. This is a teaching style which teacher preparation programs appear to be trying to eliminate!

While the task of designing effective mathematics methods courses is challenging and possibly intimidating for instructors, it is not impossible. The body of educational research is growing as educators take the lead in analyzing and improving their own profession. The studies reviewed in this chapter showed positive effects on preservice teachers in the area of attitudes and confidence, content knowledge, and pedagogical knowledge for teaching mathematics. Children will benefit from the changes teacher preparation programs are trying to make in the teaching of mathematics, especially in the area of number and operations, if beginning teachers transfer their program learning into their own classroom practices.

This chapter reviewed studies that showed not all beginning teachers' classroom practices match their personal beliefs. Not all beginning teachers' classroom practices align with the philosophies and strategies encouraged in their teacher preparation programs. The stress, steep learning curve in the first year of teaching, and the context of the teaching assignment appear to cause some to revert to teaching practices experienced in their own education. They may also be required by school administration or district policy to use programs and practices that don't match their personal beliefs or philosophies, or those of their teacher preparation programs.

This area of educational study, increasing beginning teachers' implementation of mathematics methods course mindset and pedagogy, is a smaller body of research to which this study intends to add. This study gathered and analyzed beginning teachers' perceptions about their mathematics methods courses, and how prepared they felt to teach number and operations concepts as a result of their coursework. This study used beginning teachers' feedback to study first year teaching experiences to understand which aspects of their mathematics methods coursework were transferred into classroom practice. This study also intended to examine the aspects of beginning teachers' experiences that encouraged or hindered the implementation of mathematics methods coursework philosophies and lessons.

CHAPTER 3. METHODS

3.1 Research Site

The research for this study was conducted using teachers in the United School District (USD) 383 located in Manhattan, Kansas. USD 383 has nine K-6 elementary schools, ranging in enrollment from around 300-600 students for a total of 3,500 serving a portion of three counties. Six of the nine elementary schools are considered Title I schools based on the percentage of students receiving free or reduced lunches.

The website for the U.S. Census Bureau (2010) provided the following statistics about Manhattan, Kansas, home to Kansas State University. Manhattan is near Fort Riley with a population of 52,281 for the 18.84 square mile city. The median age was reported as 27.8. Employment data showed educational services, retail trade, healthcare and social assistance, accommodation and food services, and public administration jobs resulted in a median household income of $45,992.

The eight participating teachers in this study graduated from the same university. Therefore, a brief description of the university's undergraduate and graduate program demographics is included here. According to registrar statistics published on the university's website, about 22,000 undergraduate, graduate, and veterinary medicine students enrolled at the university in the Fall of 2018. The website shows this enrollment includes 1,000 undergraduate students in the College of Education. Five of the teachers graduated with a bachelor's degree from the university's College of Education.

Education majors complete courses which include practicum experiences in block a, block b, and block c semesters. The five undergraduate participants of this study reported taking two mathematics methods courses with one being purely content based instruction, and the other course being the pedagogical methods course. The participants took the content course as part of either Block A or Block B instruction. All participants took the second methods course during their Block C instruction prior to their student teaching semester. This methods course included a field practicum placement.

The three graduate level participants in this study graduated from the university's Master of Arts in Teaching (MAT) program, a 12-month online program designed in a cohort model.

The MAT program is a part of the university's Global Campus. The graduate students take 31 credit hours over three terms from May to May. According to the university's *Global Campus Annual Report* (2017), the MAT program graduates have a 100% pass rate on teacher licensure exams upon completing the program. These participating teachers took one mathematics methods course that included a field practicum placement.

3.2 Research Design

Feelings, perceptions, and descriptions of experiences in complex contexts such as elementary school classrooms can be gathered through alternative research methods that result in personal narratives best classified as qualitative data. Hatch (2002) stated, "Qualitative studies try to capture the perspectives that actors use as a basis for their actions in specific social settings" (p. 7). Not all data is statistically quantifiable in nature, and data may be collected from fewer sources. Yet, this data represents the study participants' truths and reality, making it just as valid and important as numerical data collected from 20, 50, or 1,000 experimental trials. Qualitative studies offer a chance to analyze specific situations, which may lead to better understanding of those contexts. This can lead to improvements in particular areas such as the classroom and strengthen participants such as classroom teachers. Flyvberg (1986) wrote that, "Context-dependent knowledge and experience are at the very heart of expert activity" (p. 5). In this study, qualitative research methods produced specific data that is valuable to understanding the perceptions and experiences of beginning teachers during their first semester of teaching elementary mathematics.

This study utilized a qualitative research design to include the use of interviews for the collection of data. This study's two research questions called for the gathering of beginning teachers' perceptions of their first year of teaching elementary mathematics concepts specifically in the area of number and operations. Further interview details are included in the Research Instruments and Data Collection Procedures sections.

My interest in teacher preparation in the area of elementary mathematics led me to write the two guiding research questions. Once the research questions were set, it became clear that my data collection methods would need to utilize qualitative research methods because I wanted to study different perceptions and points of view about beginning teachers' classroom experiences. An advantage of qualitative research methods is the ability of the researcher to capture the attitudes, confidence, and perceptions about mathematical content and pedagogical knowledge in these participating teachers' own words.

An added advantage of qualitative research methods is the possibility of generating ideas for further study due to the careful analysis of the concept being studied and the context in which the study occurred. Researchers may become interested in studying a new variable or condition. Flyvberg (2006) stated that new hypotheses may be formed which lead to additional studies. For example, after reading the results of this study, the next researcher may be interested to determine whether there is a causal relationship between the timing of a mathematics methods course and the first year of teaching. Does a mathematics methods course taken in the senior year of a teacher preparation program have more or less effect on teachers' perceptions of preparedness than methods courses taken in the junior year of a program?

3.3 Research Instruments

This study utilized one-on-one interviews to better understand the experiences and perceptions of beginning elementary school teachers. Interviews are common data collection tools of qualitative research studies, and it is important to look at the particular advantages for their use. Driscoll (2011) stated, "Interviews, or question and answer sessions with one or more people, are an excellent way to learn in-depth information from a person for your primary research project" (p. 164). Alshenqeeti (2014) wrote, "this line of research seeks to explore and describe the 'quality' and 'nature' of how people behave, experience, and understand" (p. 39).

Three lists of interview questions were prepared for this study. The beginning teachers who were first year teachers participated in two interviews. Their first interview was conducted in September or October of 2018, and their second interview was conducted in December 2018 or January 2019. The beginning teachers who were in their second year of teaching were also interviewed in September or October with the set of interview questions seen in Table 1, a combination of the questions asked at both of the first-year teachers' interviews. For comparison purposes, each of the first year teachers' interview questions can be seen in Table 2.

Table 1. Interview questions for second year teachers

Second Year Teacher Interview Questions
1. Gather details about math methods course(s) and other content courses. Where? When? How long? 2. What activities do you remember from your mathematics methods course(s) in terms of content? (WHAT DID YOU LEARN ABOUT MATHEMATICS) In terms of activities & resources? (HOW DID YOU LEARN THESE THINGS ABOUT MATHEMATICS) In terms of atmosphere and culture in the class? Any exposure (& what kind) to growth mindset research? 3. On a scale from 1 to 5 (5 being most confident), how confident do you feel about teaching number and operations lessons as you started your first year of teaching? How did you decide on your rating? 4. Using the same scale from 1 to 5 did your confidence level change by Christmas break of your first year of teaching. If there was a change, ask: What do you think caused a change (if any) in your confidence level? 5. Thinking back to your first semester of teaching, what is your memory of your best lesson? What made it so good and so memorable? What would help you to teach more lessons like this one? 6. Before you started teaching your first year, what were you most looking forward to about teaching number and operations? Why? 7. Thinking back to when you had taught this lesson, how did the lessons go? (If needed prompt with: "Were the lessons what you imagined? Better? Worse?") 8. Before you started teaching your first year, what were you least looking forward to about teaching number and operations? Why? 9. Thinking back to when you had taught this lesson, how did the lessons go? (If needed prompt with: "Better? Worse? Did you plan anything special to prepare for those lessons?") 10. Describe a typical math lesson at the start of your first semester teaching. 11. Describe a typical math lesson by the end of the first semester. 12. Thinking back to your math methods course, do you think it encouraged a growth mindset about mathematics? How so? 13. During your first year of teaching, did you encourage a growth mindset in your own classroom? If yes, how so? 14. Thinking back to your math methods course(s), What aspects of the course(s), if any, improved your own content knowledge in the area of number and operations? What were the strengths and weaknesses of the course(s)? 15. Thinking back to your math methods course, what strategies, resources, or lessons did you utilized in your lessons the first semester of teaching? 16. What advice would you give instructors of math methods courses that could help ensure teachers are ready to effectively teach number and operations concepts?

Table 2. Comparison of first year teachers' interview questions

First Year Teacher Interview Questions	
First Interview	**Second Interview**
MATHEMATICS METHODS COURSE(S) DETAILS: 1. Gather details about mathematics methods course(s) from college - consider both your teaching methods and mathematics course work. Where? When? How long? 2. What activities do you remember from your mathematics methods course(s) in terms of content? (WHAT DID YOU LEARN ABOUT MATHEMATICS) In terms of activities & resources? (HOW DID YOU LEARN THESE THINGS ABOUT MATHEMATICS) In terms of atmosphere and culture in the class? Any exposure (& what kind) to growth mindset research? 3. Thinking back to your mathematics methods course, how do you think it encouraged a growth mindset? 4. What aspects of the course, if any, improved your own content knowledge in the area of number and operations? How much time was devoted to content? How in depth do you feel the content review/teaching was in the coursework? 5. Thinking back to your mathematics methods course, how do you think it encouraged the use of constructivist pedagogies? 6. What strategies, resources or lessons have you utilized in your classroom so far? 7. What strategies, resources, or lessons from your mathematics methods course do you think may prove useful as you continue to teach number and operations concepts? ATTITUDES & CONFIDENCE: 8. On a scale from 1 to 5 (5 being most confident), how confident do you feel about teaching number and operations lessons? How did you decide on your rating? Why?	ATTITUDES & CONFIDENCE: 1. On a scale from 1 to 5 (5 being most confident), how confident do you feel about teaching number and operations lessons? 2. If there is a change from the first interview ask: During the first interview, you rated yourself a ___. What do you think caused a change in your confidence rating? 3. In the first interview, you reported that your mathematics methods course did/didn't encourage a growth mindset. Did you encourage a growth mindset in your own classroom? How so? CONTENT: 4. After a semester of teaching, do you feel that your mathematics methods course provided enough content knowledge to effectively teach number and operations? 5. What were the strengths and weaknesses of the content knowledge in the mathematics methods course? 6. At your first interview, you said you were most looking forward to ____. How did the lessons go? (If needed prompt with: "Were the lessons what you imagined? Better? Worse?") 7. At your first interview, you said you were least looking forward to ___. How did those lessons go? (If needed prompt with: "Better? Worse? Did you plan anything special to prepare for those lessons?") 8. Looking back at this first semester, what is your memory of your best lesson? What made it so good and so memorable? 9. What would help you to teach more lessons like this lesson?

Table 2. (Continued) Comparison of first year teachers' interview questions

First Year Teacher Interview Questions	
First Interview	**Second Interview**
CONTENT & PEDAGOGY: 9. Thinking about your new classroom and students, what are you most looking forward to about teaching number and operations? Why? 10. What are you least looking forward to about teaching number and operations? Why? 11. Describe a typical mathematics lesson that you've taught up to this point.	PEDAGOGY: 10. Describe a typical mathematics lesson that you've taught up to this point. 11. After a semester of teaching, which philosophies and strategies taught in your mathematics methods course match up to what you experienced in your first classroom? 12. What advice would you give instructors of mathematics methods courses that could help ensure teachers are ready to effectively teach number and operations concepts?

The interview questions utilized by this study solicited specific details about the beginning teachers' preparation program specifically in the area of mathematics. This data was gathered in the first interview of first year teachers and in the only interview of second year teachers. The interview collected details about mathematics content and pedagogies introduced in the mathematics methods courses. Interview questions also asked about application of the mathematics content, and whether the beginning teacher implemented or planned to implement the pedagogies in their own classrooms.

In addition to questions about their teacher preparation program, participating teachers were asked questions about their first-year teaching practices and their attitudes towards mathematics. Some questions were adapted from Cady et al.'s (2006) study on how preservice teachers transition to experienced teachers. Cady and her colleagues asked teachers in their sixth year of teaching about their typical mathematics lesson, their favorite or best lesson and why it was chosen as the best. A question was included in the interviews to ask teachers what they would need in order to teach more lessons like their favorite lesson because the researchers specifically stated that "novice teachers have difficulty implementing recommended practices" (p. 303). The researchers also asked teachers about "changes teachers felt they had made in their practices since leaving the university and what they attributed these changes to" (p. 297).

Participating teachers were asked their opinions about what they were most looking forward to teaching, and what they were least looking forward to teaching in number and operations during their first year of teaching. These questions were adapted from Robinson and Adkins (2002) study of preservice teachers' attitudes towards mathematics. The researchers were examining the effects of a mathematics methods course that was designed to include "manipulatives, hands-on materials, cooperative learning, and technology" (p. 19) through small group discussions and activities. The instructors of the methods course designed the course to be "a constructivist approach to learning and doing mathematics" (p. 19).

This study utilized these two questions to determine teachers' attitudes towards topics they were most and least looking forward to teaching. This study intended to gather perceptual data about how the lessons went when they were eventually taught, and whether the teachers' attitudes changed towards the mathematics concept after their lessons. This researcher wanted to determine if there would be a connection between the concepts they were most or least looking forward to teaching and their most memorable lesson by the end of the first semester of teaching.

The interviews were audio-recorded and transcribed for analysis as suggested by Alshenqeeti (2014) so that the "researcher could record the respondent's words with greater accuracy than that achieved through obtrusive and disruptive 'on the spot' note-taking" (p. 43). The set of interview questions used at each type of interview have been charted separately and included in the appropriate sections of Chapter 4. This is to improve readability and to show comparisons between the interviews conducted that semester. For example, in Table 2, the two first year teacher interview questions are shown in a side by side format to show connections between questions that were designed to determine what changes, if any, occurred in beginning teachers' perceptions or experiences through the first semester of teaching. In Table 2, the questions are also grouped by the categories of mathematics methods course details, attitudes and confidence, content knowledge, and pedagogy.

Table 4 shows the questions first asked of all participants at the start of their interviews to gather information about their mathematics methods coursework. These questions were asked in the September and October 2018 interviews. Tables 5, 7, and 8 are separate charts which show the questions asked of each participant in the three areas of attitudes and confidence, mathematical content knowledge, and mathematical pedagogical knowledge. These tables are also included in the appropriate sections of Chapter 4.

3.4 Participants and Participant Selection

Participating beginning teachers in this study were chosen based on specific criteria. For the purpose of this study, beginning teachers were considered first and second year teachers. A list of first- and second-year teachers in the school district was generated, and participants were recruited either in person or by email. For inclusion in this study, the participants needed to be assigned to teach elementary mathematics during their first year of teaching, and they had to be enrolled in at least one mathematics methods course during their teacher preparation program.

Eleven teachers originally agreed to participate in this study. However, three teachers were not included because they did not meet the qualifications for the study either because of their assigned teaching duties, or because they did not take a mathematics methods course in their teacher preparation program. Of the eight participating teachers, four are first year teachers, and four are second year teachers. A total of 12 interviews were conducted.

The eight beginning teachers who participated in this study are female, elementary teachers assigned to teach elementary mathematics. The first-year teachers are K-3 teachers, and the second year teachers are K-4 teachers. All eight of the participating teachers attended the same local college's teacher preparation program to earn either an undergraduate or graduate degree. None of the participants were in classes together. Three of the eight teachers participated in a one year Masters of Arts in Teaching program. In this program, the mathematics methods course was an online course for either eight weeks (one teacher) or for a full semester (two teachers). Table 3, on page 58 of this paper, charts basic data gathered during the interviews about each beginning teachers' preparation program and grade level assignment for the first year of teaching elementary mathematics.

3.5 Data Collection Procedures

The collection of qualitative data must be organized and analyzed which can potentially pose challenges to the researcher. Visser, Krosnick, and Lavrakas (2000), for example, noted that the data is likely to be just as diverse and unpredictable as the people who made up the study's sample population. In fact, the authors shared an example of this diversity by discussing possible answers to the question, "What is the most important problem facing the country today?" (p. 237). They wrote, "In a survey of 1,000 respondents, nearly 1,000 different answers will be given if considered word-for-word" (p. 238). This section describes and justifies the research data collection procedures utilized in this educational study in order to maximize the advantages and minimize the limitations of qualitative research methodology.

During the first semester of the 2018-2019 school year, beginning teachers were interviewed about their attitudes and confidence concerning the teaching of number and operations concepts. Those beginning teachers who are first year teachers were interviewed twice with two different sets of questions. They were interviewed at the start and end of the first semester of their teaching career. Those beginning teachers who were in their second year of teaching were interviewed once using a combination of both sets of interview questions.

The goal of the interviews was to determine beginning teachers' perceptions about their preparation in terms of mathematical content and pedagogical knowledge. They were also asked how they felt their mathematics methods course(s) in their teacher preparation program prepared them to teach number and operations concepts in their first year of teaching. Interview questions asked participants to report their own level of understanding mathematical content in the area of number and operations before and after taking their mathematics methods course(s). Through open ended questions, participants were also asked to report examples of constructivist pedagogy learned through their mathematics methods course(s) which they utilized or intended to utilize when teaching number and operations concepts in their own classrooms.

Participants met with the researcher for one or two 45-60 minute, semi-structured interviews. According to Alshenqeeti (2014), a semi-structured interview is an interview that initially utilizes a set of predetermined questions, but allows interviewer the flexibility to probe into interviewees' responses to gain or clarify information. This helps to both ensure that the researcher understands the interviewee's intended answers, and that the interviewee understands the purpose of the questions being asked.

These semi-structured interviews were used to first gather perceptual information about participants' attitudes toward teaching number and operations concepts. Then the participants were interviewed to determine what changes, if any, occurred in the perceptions they expressed at the start of the school year. Interview questions asked whether there was a change to their feelings about their teacher preparation program in general, and their mathematics methods course(s) in particular. Did they still feel that their content and pedagogical knowledge was or was not impacted by their mathematics methods course(s) in their teacher preparation program? Did their attitudes towards teaching mathematics in general and confidence in their ability to teach the number and operations concepts change through their first semester?

Demographic information about the participating teachers and their classrooms was collected at the first interview. The data describing the participating teachers included the name of their teacher preparation program, number of mathematics methods courses completed, and the year(s) of college in which they attended the mathematics methods course(s). Teachers were asked to share information about other mathematics content courses they had taken. Teachers were asked to report their grade level assignment during their first year of teaching mathematics.

Teachers were asked about their own attitudes toward mathematics in terms of positivity and to rate their level of confidence in teaching number and operations concepts at the start and end of their first semester of teaching elementary mathematics. Johnson and Morgan (2016) stated that, "In some instances, the researcher might develop a survey scale in which respondents self-report perceptions about their knowledge level" (p. 65). This study used the five point scale, with five being the most confident, to gather data about the teachers' initial feelings of confidence to determine what changes, if any, occurred in their confidence level as the semester progressed. Teachers were also asked how they determined their rating, and what might have caused any change in their confidence by the end of their first semester of teaching.

The interview questions were written to solicit information directly related to the three sections of the two research questions. Those sections are attitudes and confidence, content knowledge, and pedagogical knowledge. The questions are listed in an order based only on the direction the researcher anticipated the conversation to take, but reordering of the questions did occur in reaction to participants' responses to questions.

The interviews were held at a location chosen by the participating beginning teacher in order to provide a comfortable, confidential setting for the participants. An interview journal was kept by the researcher for recording names, dates, and times of interviews as well as general impressions the researcher wished to capture for future reflection. The interviews were audio recorded to help the researcher accurately capture the participants' exact words and phrases. The recordings were then transcribed within 24 hours of the interview for analysis and deleted at the end of the study. Each interview lasted between 45-60 minutes.

3.6 Data Analysis

Erickson (1986) stated, "The task of the analyst is to uncover the different layers of universality and particularity that are confronted in the specific case at hand - what is broadly universal, what generalizes to other similar situations, what is unique to the given instance" (p.130). This section describes the procedures that guided the typological analysis of qualitative data from a total of 12 interviews over a six month period during the first semester of the 2018-2019 school year. The interviews were conducted during the months of September, October, and December 2018 or January 2019. The interviews were analyzed using typological analysis procedures in the months directly following each set of interviews.

In order to organize the data from each interview, a naming system was developed to be used as a code to ensure confidentiality of the participants while allowing the researcher to keep track of the data pieces. In a confidential research log a list of participants was written in the order of which the first interviews took place. The participants were then assigned a letter from A through H. The dates and times for each participants' interviews were recorded in the research log as well as this researcher's notes which included personal impressions formed during the interviews. Table 3 summarizes the information gathered from teacher interviews in this study.

The codes I1 and I2 were used to name data from the two interviews. The pages of each transcribed interview were numbered and included in the coding of data pieces. Therefore, a piece of data from the third page of Teacher D's second interview will be coded as DI2-3. As the typological analysis was completed, a code was written for each data piece to notate where the data originated as suggested by Hatch (2002).

Table 3. Participating teachers' data

Teacher	Teaching Degree Earned	Mathematics & Mathematics Methods Coursework Taken *During* Teacher Preparation Program	Grade Level Assignment	Year of Teaching
A	Bachelor's	Math for Elementary Teacher & mathematics methods course	1st	First
B	Bachelor's	Math for Elementary Teacher & mathematics methods course also had mathematics concentration	K	First
C	Bachelor's	Math for Elementary Teacher & mathematics methods course also had mathematics concentration	K	Second
D	Bachelor's in Broadcast Journalism; Master's in teaching	Teaching Elementary Mathematics methods course	1st	First
E	Bachelor's in Family Studies & Human Services; Master's in teaching	Combination mathematics & science methods course	1st	Second
F	Bachelor's in Chemistry; Master's in teaching	Teaching Elementary Mathematics methods course	K	First
G	Bachelor's	Geometry for Teachers; mathematics methods course	3rd	Second
H	Bachelor's	College algebra, probability & statistics; mathematics methods course	4th	Second

Further details of the typological analysis procedures followed by this study are necessary. Hatch (2002) described typological analysis as a process for analyzing qualitative data that "starts by dividing the overall data set into categories or groups based on predetermined typologies" (p. 152). The predetermined typologies, or themes, used in this study were the components described in the two research questions. Therefore, the data was first sorted using three different colored highlighters to identify items matching the three categories of attitudes and confidence, mathematical content knowledge, and mathematical pedagogical knowledge. These are the three categories embedded within this study's two research questions. Each colored data piece was coded using the system previously described, and copied and pasted into a separate document for each of the three categories.

The next step in this study's data analysis was to look for patterns, relationships, and emergent themes within each of the three categories. Hatch (2002) defined patterns as “regularities” and relationships as “links” in the data pieces (p. 155). She suggested a two-step process for this part of the data analysis. First, she suggested writing a brief summary statement, or generalization, for each excerpt taken from interviews. She justified this step by saying, “Expressing findings as generalizations provides a syntactic device for ensuring that what has been found can be communicated to others” (p. 159). The second step was to resort the data into chunks and display the data in a way that illustrates the common patterns, relationships, and themes that emerged from the surveys and interview excerpts. A sample sorting chart used during this study’s typological analysis shows coded data pieces taken from transcribed interviews, sorted into thematic categories (see Appendix).

The last stage of the typological analysis used in this study was to specifically analyze the nonexamples, or uncoded data, which did not fall into one of the three predetermined themes identified by the research questions and their components. Hatch (2002) discussed two purposes for this stage of analysis. The first purpose of this analysis was the determination of whether or not the categories themselves are “justified by the data” (p. 157), the findings within the three categories. The second purpose was to determine whether the uncoded data is contrary to the findings of the coded excerpts, or merely new and different perceptions or experiences of beginning teachers. Any new classifications of data or data that lead to new patterns, relationships, or themes are shared in the findings of this study.

The typological analysis described in this section provide organization for data collection. The research log and coding system were designed to help ensure the researcher had a systematic approach to gathering and representing data pieces while still preserving the confidentiality of the participants and the integrity of the data itself. Interview data was analyzed in an ongoing manner throughout the course of the study. Data excerpts were classified, color-coded, and generalized based on the research questions that led this study. Within each of the three classifications, patterns, relationships, and themes were identified and summarized for reporting. Data that did not fit into the predetermined categories was also analyzed and reported.

3.7 Limitations

Erickson (1986) wrote “…it seems that there is so much variation across classrooms, and so much variation in the implementation of "treatments" themselves that large-scale program evaluation by quasi-experimental methods is very problematic" (p. 131). As Erickson implied, experimental and quasi-experimental methods are not appropriate for all contexts, nor are they appropriate for all purposes of educational studies. In this study, there was variability in the experiences of the cooperating beginning teachers. While all of the participants attended the same local college, their teacher preparation program and mathematics methods course experiences, assigned schools, and assigned grade levels differed.

Since the purpose of this study was to understand not manipulate the current experiences of a relatively small number of beginning teachers, an experimental, quantitative methodology design was not appropriate. This study was designed to collect and analyze teacher perceptions of how prior events such as mathematics methods coursework did or did not affect their preparation for their first year of teaching. Therefore, this study was designed using the qualitative research method of semi-structured interviews despite some recognized limitations. The limitations of qualitative research methods such as interviews can be grouped into two main categories. First is the inability of researchers to offer broad generalizations as a result of a qualitative study. The second category of limitations revolves around the researcher themselves.

Hatch (2002) noted that qualitative research methods are frequently used in studies with fewer cases. Sometimes this is because the concept or context being studied is a rare phenomenon, and sometimes the small case number is a simple matter of economics or convenience. Regardless of the reason for the small case number, one limitation of these studies is the inability to generalize findings to a larger population or context. For example, even after careful analysis of the collected data, a reader cannot assume that all beginning teachers' perceptions are the same as the perceptions shared by the eight teachers interviewed in this study.

Determining causal relationships between variables from the findings of this study and similar, small qualitative research studies is also problematic, and it is not the goal of qualitative inquiry. For example, analysis of this study's data may seem to show a cause-effect relationship between two variables when it may be that a third unreported, unobserved variable triggered the outcome. In addition, a causal relationship found in one case would not be able to be generalizable to a larger population. The information and data gathered will be descriptive in nature, but will not be predictive of future relationships or results.

The second broad group of limitations for qualitative research methods such as interviews is directly related to the ethics, integrity, sensitivity, and expertise of the researchers themselves. Stake (2010) cautioned that the quality of the study's design, data gathering, and data analysis are all affected by the researcher. Since a single person in this study is recruiting participants, gathering, and analyzing the collected data, the researcher must remain an objective, unbiased observer and reporter of data. Researchers must be careful not to seek answers that support their own personal opinions. As a 25 year teaching veteran with interest in becoming a mathematics methods instructor, I do have opinions about what should be included in a methods course. This is a personal bias that I had to be conscious of during my interviews.

The researcher must also be skilled in creating an atmosphere in which the interviewee feels comfortable. However, it must be recognized that, as Alshenqeeti (2014) stated, "interviewees will only give what they are prepared to reveal about their perceptions of events and opinions" (p. 43). This study's results include unconfirmed facts since it was assumed that the participating teachers answered the interview questions honestly, and that their answers were not influenced by the researcher.

The strengths and weaknesses of qualitative research methods such as interviews were considered during the planning phases of this study. In an article addressing misconceptions about qualitative research methods, Flyvberg (2006) mentioned two valid and compelling reasons to choose qualitative methods despite possible limitations. The author specifically addressed critics of qualitative research methods who question the value of conducting studies with single or small case numbers. He specifically addressed the generalizability misconception. First he stated, "Predictive theories and universals cannot be found in the study of human affairs. Concrete, context-dependent knowledge is therefore more valuable than the vain search for predictive theories and universals" (p. 7). Second, Flyvberg warned that the problems with generalizing or summarizing findings from qualitative research is "due more to the concept being studied than to the research method itself" (p. 25).

In this study, teachers' perceptions of how prepared or unprepared they were to teach number and operations is a difficult concept to generalize, but the methods used to collect and analyze the perceptions are valid. During the design process of this study, decisions were made to try to maximize the strengths and reduce possible weaknesses as much as possible.

3.8 Summary

Qualitative research methods were chosen for this study. As Starman (2013) noted, qualitative methods are valuable for gathering data in practice-oriented fields such as education where the information collected can focus on the study participants' reactions to and perceptions of their environment and context. The methods chosen for this study were designed to help teachers reflect on their own feelings and practice which has the potential to be an additional personal benefit to the study's participants.

Interviews were chosen in agreement with Cochran-Smith, et. al. (2015) who wrote that "small-scale, mostly single-site studies contribute important insights to the field by theorizing complex aspects of teacher preparation practice" (p. 117). The findings from this study could potentially help mathematics methods course instructors utilize the perceptions and memories of the first year of teaching to "provide opportunities for analysis and discussion" (Lomas, 2009, p. 18) concerning ways to improve aspects of teacher preparation programs.

The interview questions were written to coincide with the three components of the two research questions. The questions were written to encourage participants to share their perceptions and experiences so that I can build a better understanding of the participants' first year of teaching and how it was or was not affected by their mathematics methods course(s). The semi-structured format of the interviews gave this researcher freedom to investigate participants' responses and comments.

CHAPTER 4. RESULTS

The eight participating teachers, four first year and four second year, in this study attended the same university over a three year period for either undergraduate or graduate level elementary teacher preparation. Five earned undergraduate teaching degrees, and three attended the university's graduate program for a Master's degree in teaching. Regardless of the program, this university offered a single mathematics methods course although each of these eight teachers' courses had different instructors. Table 4 on the following page shows questions asked of each participant to gather information about the teacher preparation program, mathematics content course(s), and the mathematics methods course. For seven of the eight participants, the course was a full semester. The five undergraduate participants took the course live versus the online offering for the graduate level participants. For the undergraduate program, there was also a mathematics content course called Mathematics for Elementary Teachers. This course was offered and taught through the university's mathematics department. Three of the five teachers who earned undergraduate teaching degrees took additional college mathematics courses, and two of those teachers had mathematics as their concentration area.

All eight teachers were interviewed at the start of the 2018-2019 school year. The four first year teacher participants were also interviewed in December 2018 or January 2019, the end of their first semester of teaching elementary mathematics. In this chapter, data from the interviews is shared in text and tables to note thematic patterns in the evidence regarding the two main research questions. Under both research questions, there were subsets of questions to narrow the focus of teachers' perceptions into the three areas of attitudes and confidence, mathematical content knowledge, and mathematical pedagogical knowledge. The remainder of this chapter is organized by these three subsets of questions.

The first interview's focus was to determine beginning elementary teachers' perceptions of how well their mathematics methods course(s) prepared them for their first year of teaching number and operations in terms of their attitude and confidence, content knowledge, and pedagogical knowledge. The second research focus of the interviews was to determine what changes, if any, there were in the beginning elementary teachers' perceptions in each of those areas by the end of their first semester of teaching.

Table 4. Interview questions concerning mathematics methods coursework details

First Year Teacher Interview Questions	Second year Teacher Interview Questions
First Interview	**Only Interview**
MATHEMATICS METHODS COURSE(S) DETAILS: 1. Gather details about mathematics methods course(s) from college - consider both your teaching methods and mathematics course work. Where? When? How long? 2. What activities do you remember from your mathematics methods course(s) in terms of content? (WHAT DID YOU LEARN ABOUT MATHEMATICS) In terms of activities & resources? (HOW DID YOU LEARN THESE THINGS ABOUT MATHEMATICS) In terms of atmosphere and culture in the class? Any exposure (& what kind) to growth mindset research? 3. Thinking back to your mathematics methods course, how do you think it encouraged a growth mindset? 4. What aspects of the course, if any, improved your own content knowledge in the area of number and operations? How much time was devoted to content? How in depth do you feel the content review/teaching was in the coursework? 5. Thinking back to your mathematics methods course, how do you think it encouraged the use of constructivist pedagogies? 6. What strategies, resources or lessons have you utilized in your classroom so far? 7. What strategies, resources, or lessons from your mathematics methods course do you think may prove useful as you continue to teach number and operations concepts?	MATHEMATICS METHODS COURSE(S) DETAILS: 1. Gather details about mathematics methods course(s) from college - consider both your teaching methods and mathematics course work. Where? When? How long? 2. What activities do you remember from your mathematics methods course(s) in terms of content? (WHAT DID YOU LEARN ABOUT MATHEMATICS) In terms of activities & resources? (HOW DID YOU LEARN THESE THINGS ABOUT MATHEMATICS) In terms of atmosphere and culture in the class? Any exposure (& what kind) to growth mindset research? 3. Thinking back to your mathematics methods course, how do you think it encouraged a growth mindset about mathematics? 4. Thinking back to your mathematics methods course(s), What aspects of the course(s), if any, improved your own content knowledge in the area of number and operations? How much time was devoted to content? How in depth do you feel the content review/teaching was in the coursework? What were the strengths and weaknesses of the course(s)? 5. Thinking back to your mathematics methods course, how do you think it encouraged the use of constructivist pedagogies? 6. Thinking back to your mathematics methods course, what strategies, resources, or lessons did you utilized in your lessons the first semester of teaching?

4.1 Research Focus - Attitudes and Confidence

Teachers were asked for details about their attitudes and confidence for teaching mathematics and how it may or may not have been influenced by their mathematics methods coursework. Teachers were specifically asked for details about how growth mindset may or may not have been introduced, modeled, or encouraged by their mathematics methods coursework. Teachers were asked to report whether they had implemented aspects of growth mindset into their own classrooms and to give examples of their classroom practices that align with a growth mindset. Teachers were asked to report whether there were changes in their attitudes and confidence by the end of the first semester of teaching, and, if applicable, to share their perceptions about what may have caused the changes.

The data gathered in the area of attitudes and confidence is discussed in the following two sections. In the first section, beginning teachers' perceptions of mathematics in general and of their methods courses and instructors are shared. Then the beginning of school perceptions of attitudes and confidence are shared. Second, the end of semester perceptions of attitudes and confidence and possible changes in each are shared.

4.1.1 Perceptions at Beginning of the Year

The teacher interviews began with questions about their own mathematics education, background, and classes as potential evidence about their attitudes towards mathematics. It was assumed that students who enrolled in more mathematics courses most likely had more positive attitudes towards mathematics. Many of their answers seemed to form three general categories which have been entitled, "Math is hard," "I'm teaching math in a different way from how I learned math," and "I'm comfortable with math because of my background or emphasis."

Although six teachers made reference to mathematics being either a tough, hard, or even terrifying subject that could cause struggle and frustration for the teachers themselves, only Teacher G remembered talking about mathematics anxiety in her methods course. Most of the teachers referenced either their own struggles with mathematics, or their peers' struggles. For example, Teacher E told of growing up with a disability in mathematics that led her to choose a degree program and career with the least amount of mathematics. It wasn't until after ten years of teaching preschool that she felt confident enough to go back to school for elementary education. Her experiences in her mathematics methods course and second grade practicum changed her attitudes about mathematics to such a degree that mathematics is now her favorite subject to teach. She reported, "So I've always hated, hated math, and I've been terrified of it. I was really afraid and anxious when it comes to math. I wanted to be a teacher, but I did not want to do anything that had to do with math. That has since changed."

When Teacher E was assigned her second-grade practicum in coordination with her mathematics methods course she reported, "I was really scared. I had so much anxiety and really upset and crying because I just kept thinking, I'm never going to be able to teach second grade math." Teacher E completed an eight week long, graduate level mathematics methods course online. She credited the experiences she had in this class and her practicum placement for the changes in her attitude towards teaching and learning mathematics. She reported,

> But as I kept going and my learning in this class ways to have fun with the kids teaching math because if they're not having fun they're not learning with math. I mean, it can get boring if you're not having them do stuff. The teacher I had, he had tons of fun activities. I was thinking, well the kids are learning and they're playing, but they're learning. I started to get a little more comfortable. By the time I was fully teaching however long you do in your internship, I actually was really enjoying math. This was really fun seeing the kids connect, and now it's my favorite subject to teach. I love it. I love teaching math. I do it first thing in the morning so I'm always just really excited to get going and I think the kids can tell I'm excited to get going. I have tons of math games and manipulatives and ideas. Hands-on things and group things.

Two teachers, Teachers B and D, mentioned that they experienced or witnessed frustration in their university mathematics methods courses. When asked if anyone ever got frustrated, Teacher D said, "Yes. Math is tough." Teacher B's mathematics methods course instructor utilized constructivist activities where the preservice teachers had to draw out or model solutions with manipulatives in what Teacher B called "inquiry-based math." When asked why she thought people got frustrated during these activities Teacher B said, "I think this was the first time for a lot of students to like really discover why they do it (the algorithm), so people did get frustrated or would try to go straight to the algorithm."

Two teachers, Teachers G and H, named specific mathematics concepts that were weaknesses of their own. These weaknesses were reportedly why they were least looking forward to teaching those concepts to their own students their first year of teaching. Teacher G commented about how she didn't use tangrams her first year of teaching fractions because "Fractions also scared me. It would probably be my second least favorite after measurement." Teacher H also reported that fractions was the area of mathematics she was least looking forward to teaching because, "It was just one of my weaknesses growing up and we didn't really touch base on it in blocks, and I didn't get to see it during interning."

The second category of responses could be entitled, "I'm teaching math in a different way from how I learned math." Five of the eight teachers made reference to how different their own mathematics education was from how they were being prepared to teach the subject. Teacher F stated, "When I was growing up in South Korea everything was just book and worksheet, book and worksheet. You never get to work with manipulatives in class, I think, up to my year in 12th grade experience." In addition to the previous comments about games and hands-on activities she uses today, Teacher E noted that, "I wish I was a kid now in school by how much kinder and understanding teachers are."

Teachers A, B, and D commented on the algorithmic, process knowledge that was emphasized in their own education. Teacher A commented that, "Growing up I always knew the process on how to borrow or carry or whatever we were doing, but I didn't always understand the actual math behind it." Teacher B said, "When we went through elementary school we were just handed the algorithm." Teacher D said, "When I was growing up doing math it was like this is how we do it. This is the answer you should get. Everything else is wrong."

Three of the four first year teachers and one of the four second year teachers reported completing either multiple high school, dual credit courses or university level mathematics courses. Because of this pattern of responses, a third category was created for statements teachers made about being comfortable with mathematics content because they had taken many content courses. For example, Teacher A reported completing a college algebra, trigonometry, and statistics course in high school for dual credit. Therefore, she only took two mathematics courses, to include the mathematics methods course, in her teacher preparation program.

Teachers B and C took 15 credit hours for an emphasis in mathematics during their teacher preparation program. Teacher F earned an undergraduate degree in chemistry and nutrition which she stated included multiple mathematics courses.

Statements from three teachers fell in the area of attitudes in mathematics, but did not fall into the three previously mentioned categories. Teachers E, G, and H all mentioned experiences with high school or university mathematics content courses. Teacher E mentioned choosing a degree that required no mathematics coursework. Teacher G chose English as her area of emphasis for her teacher education program because it was her strength in high school. She also commented that she wished she had chosen mathematics as her emphasis because, "The people that emphasized in math got more math classes. I don't know if it was more methods, you know. I don't know how much better it was for them, but it definitely would have helped a lot. That's why I feel more comfortable with literacy just because I took so many more classes."

Teacher H reported taking the minimum number of courses in high school mathematics, and then she experienced university level mathematics courses with large student enrollment. She reported,

> My algebra class, it was lecture type style and there was probably 300 kids in that lecture. And they break it up, I forget what it was called, but they break it up so you have smaller classes and go over the homework and stuff like that in a smaller environment. My statistics was about 40-50 and we were in computer labs and did a lot of it on the computer.

At the start of the 2018-2019 school year, the eight teachers were asked to use a five-point scale, with five being the most confident, to report how confident they were to teach number and operations concepts their first year of teaching. Table 5 shows the specific questions asked of each participant in the area of attitudes and confidence. Numerical responses ranged from a one to a four. When asked to explain how they determined their answer, the responses were based on concerns that could be grouped into three categories, concerns based on their perceived knowledge of mathematical content, their knowledge of students at a particular grade level, and their knowledge of mathematical pedagogy.

Table 5. Interview questions concerning attitudes and confidence

<table>
<tr><th colspan="2">First Year Teacher Interview Questions</th><th>Second Year Teacher Interview Questions</th></tr>
<tr><th>First Interview</th><th>Second Interview</th><th>Only Interview</th></tr>
<tr><td>ATTITUDES & CONFIDENCE:
1. On a scale from 1 to 5 (5 being most confident), how confident do you feel about teaching number and operations lessons? How did you decide on your rating? Why?</td><td>ATTITUDES & CONFIDENCE:
1. On a scale from 1 to 5 (5 being most confident), how confident do you feel about teaching number and operations lessons?

2. If there is a change from the first interview ask: During the first interview, you rated yourself a ___. What do you think caused a change in your confidence rating?

3. In the first interview, you reported that your mathematics methods course did/didn't encourage a growth mindset. Did you encourage a growth mindset in your classroom? How so?</td><td>ATTITUDES & CONFIDENCE:
1. On a scale from 1 to 5 (5 being most confident), how confident do you feel about teaching number and operations lessons as you started your first year of teaching? How did you decide on your rating?

2. Using the same scale from 1 to 5, did your confidence level change by Christmas break of your first year of teaching? If there was a change, ask: What do you think caused a change (if any) in your confidence level?

3. During your first year of teaching, did you encourage a growth mindset in your classroom? If yes, how so?</td></tr>
</table>

Teacher H and Teacher G based their confidence ratings on their knowledge of mathematical content and students at a particular grade level. Teacher H rated herself a one or two because she reported thinking, “Oh, man I’m going to have to do all this by myself. I haven’t done fractions in how many years. I haven’t seen any of the curriculum so I was just I’m seriously going to have to reteach myself all of this. I just haven’t seen it. I haven’t seen how the kids will do, what they did in third grade.” Teacher G rated herself a 4 or high 3 because she felt she “was still pretty confident as a first year teacher just because I had done my internship in the same grade level. I had already done the same things with my teacher last year that I was going to be doing by myself this time.” Teacher G did note, “If I’d been at a different grade level I’d definitely have been a 1 or a 2 because I would have felt lost.”

For three teachers in this study, lacking knowledge of students was the sole concern used to rate their confidence to teach number and operations concepts. For Teacher C, not having experience with kindergarten caused her to rate herself a one at the start of the school year. Teachers E and D were also concerned with the abilities of their first-grade students. Teacher E worried about "what kind of learning disabilities they were going to have," and Teacher D didn't feel like she was "prepared for was being in a Title I school and having kids that are low."

The remaining three teachers, Teachers A, B, and F, were comfortable with their mathematical knowledge and had some experience already with the grade level assignment for their first year of teaching. Their area of concern was whether or not they would have the mathematical pedagogical knowledge to effectively teach the mathematics concepts. For example, Teacher A who rated her confidence a 3.5 or 4, stated,

> Once you get to the regrouping and borrowing that's when I think it gets challenging because I know how to do it, but explaining it is like a whole nother ball game. Some of the chapters where it gets a little more complex, I might need to think of other resources and really sit down and look at it more and figure out how I can give the information to them in a way that they're going to need to truly understand it.

Teacher B rated her confidence a 3. She also made statements of concern about her mathematical pedagogical knowledge. Teacher B stated that she had hopes of creating lessons that used mathematical discourse and discovery, but doubted her ability. She stated,

> I feel least prepared where we're going to transition into multiplication next and I think I feel least prepared skills-wise, like teaching strategy-wise, on that one. To really start it off inquiry-based so they are discovering why and what multiplication means. To really facilitate good discussions with introducing multiplication and division, I don't feel like I have a ton of skills on that.

Teacher F rated her confidence a 4 because she felt comfortable with the content and age group she was assigned to teach her first year. She was even comfortable with a lot of the mathematical pedagogies needed in her kindergarten classroom. She was used to teaching small groups of early childhood students using hands-on manipulatives when she worked as a paraprofessional in a Montessori school. Her area of pedagogical concern dealt specifically with teaching whole group lessons and staying on pace with her kindergarten colleagues. Here is an example of her specific concerns:

> How am I going to give them enough time to catch that up? If they don't catch up, yes they will have some time to catch up in first grade, but then first grade content will be slowing down there. There's not enough time for me to have one on one connection with them to see how they're progressing. That's still my weak point, I have to say. I am used to teaching one on one.

Next teachers were asked whether they had exposure to growth mindset research, and how they felt their mathematics methods course encouraged and incorporated growth mindset. Their responses fell into two categories. The first category included four teachers who felt that growth mindset was definitely included in their methods coursework although one teacher had to be prompted with an example of growth mindset. The second category included the other four teachers who felt that although the term growth mindset wasn't specifically included in their coursework, aspects of growth mindset were encouraged.

Teachers A, D, and F remembered specific lessons about growth mindset in their mathematics methods course, and they commented on the benefits of growth mindset. For example, Teacher A said that the methods instructor warned the preservice teachers that, "you might not like my class because I'm going to make you work hard and really break this down and think about how to teach your kids." Teacher A said the class had numerous discussions about how important it was to not give up.

Teachers D and F remembered their growth mindset lessons focusing on changing their attitudes towards handling student mistakes. Teacher D remembered her instructor telling the class, "The most successful kids in math are the kids that have been taught that they're allowed to make mistakes, and that mistakes is part of that growth." Teacher F remembered learning to word things differently when talking with students about mistakes. For example, "instead of saying you're wrong so you're getting a point off say, 'You've done two correct. Let's do better next time.' It will help the children have a positive experience in a math lesson and it will make them do better next time."

When asked about growth mindset in her methods course, Teacher E initially asked, "What would be an example of that?" After a brief statement about growth mindset, the teacher immediately agreed it was included in her mathematics methods course. She shared specific details of classroom talk and practices that were taught and encouraged in her course. For example, she never uses the phrase "It's so easy" in her classroom, and she discourages her students from using it also. She feels that is "detrimental to children who that's not easy for."

Teacher E described specific growth mindset grading and assessment practices introduced in her methods course that she has implemented in her classroom. She was taught to analyze student work to specifically look at strategies and methods. She said, "The way that they taught us to grade is to not necessarily just look at 4 + 1 is 6 and saying the whole thing is wrong. They missed that. It's looking to see the different ways, how do they get to the problem. Look at the different ways of solving it." Teacher E shared a growth mindset grading practice that focused on what the student could do, versus what they hadn't completed. She explained, if the child:

> Only got half of the test done, but I saw they were working hard, and they were persevering, and they were doing their best, I'm just going to not grade them for the stuff they didn't do and only grade them for the stuff they did do. They taught us a different way of grading to reward the children for the hard work that they do.

The other four teachers, Teachers B, C, G, and H, felt that growth mindset aspects were encouraged, but the term was not specifically mentioned in their mathematics methods course. Teacher B felt it was mentioned in other classes in her teacher preparation program, but gave no specific examples or memories of those lessons. Teachers C and H felt that encouraging teachers and students to try different methods and strategies was an example of a growth mindset practice. Teacher H felt the instructor encouraged this practice "to get the kids to keep trying to find ways that work for them" while they're learning mathematics concepts.

Teacher G felt that her mathematics methods instructor encouraged aspects of growth mindset by emphasizing how to talk with students about mistakes. She remembered the instructor telling the class that they should tell students, "You're not always going to get stuff right." She also remembered her instructor saying she should "show them that it's ok to make a mistake. Purposely mess up and make them correct you. Show them it's ok for that to happen and or to correct each other." Teacher G stated, "I know that growth mindset is more than just 'It's ok to be wrong.' That's what I remember, what I relate to growth mindset."

Next, teachers were asked the extent to which they might have implemented growth mindset in their own classrooms during their first months of teaching. They were encouraged to share examples of classroom talk and practices they felt demonstrated aspects of growth mindset. Two large themes were evident from the teachers' interview comments. First, teachers handled mistakes and frustration with growth mindset talk either directly or indirectly. Second, teachers' comments revealed how they encouraged effort and working hard in their classrooms.

Six teachers talked about how they handled student mistakes and frustration as an example of how they encouraged a growth mindset in their classrooms. However, only Teacher A specifically used phrases associated with Dweck's (2006) writing on growth mindset. Teacher A mentioned, "We've done one lesson on like how we can't say 'I can't do this.' It's 'I can't do it *yet*.'" Teacher A also talked with her students about how their brains were growing when they did difficult work. She felt the students were starting to take on the growth mindset because she'd heard them saying things like, "Yeah, my brain worked really hard today." Teachers B, D, E, F, and G also felt they encouraged a growth mindset by how they handled students' frustration during mathematics class. Below are some examples in the teachers' own words:

- Teacher D - "What do I do? Getting them comfortable with being confused and giving them strategies. If you're confused, do this. If you're still confused, do this. Giving them some specific concrete things to do when their first method doesn't work." Teacher D also mentioned her concern for how students talked about their mathematics ability. She stated, "Kids in first grade have already said, labeled themselves, 'I'm bad at math.' Like, who told you that? How do you know that? Why do you think you're bad at math?"
- Teacher E - "So even if it might not be true for one of my kids in my class, I still say it's ok. I'm not mad. You will get this. Oh, I'm stupid. No, you're not. You're still learning. I always say, this is why you're in school. You're in school to learn things. That's why I'm here. I'm here to show you how to do it, and teach you how to do it. I just keep

re-explaining it as many different ways as I can possibly come up with. I think I spent 30 minutes on Thursday just teaching what number comes before 7."

- Teacher F - "Just in between lessons, or in between the work, if they're frustrated I'll just say it's ok. You can do your best and show me you can do your best. That's all I want to see."

Teachers B, D, E, and F felt that how they encouraged effort and hard work were examples of growth mindset in their classrooms. The following quotes from teacher interviews show examples from their classrooms:

- Teacher B - "Ideally, I don't know how well I've done this so far, but ideally it would be, I try to celebrate guessing, so just the goal is effort not the end result. Celebrating different strategies a lot more so than right answers."
- Teacher D - "I'd rather see a kid who is working, working, working past obstacles. Oh, that didn't work, I'm going to try something else. Who's got that grit? That person to me is showing way more growth than a person who sits down and gets it right and they're done. So training your students that...I tell my students that the harder it is, that's your brain getting stronger, you know. If we're doing easy things all the time, your brain isn't getting stronger. I tell them their brain is growing. To teach them what to do when something doesn't work, or when an answer isn't right. Being able to get over that obstacle and embrace that as part of the journey."
- Teacher F - "We haven't really talked the title called growth mindset, but the very first math class, when a student really didn't understand how to do the workbook pages, the assignment that I'm giving them, I'm teaching them just do your best. There's no wrong or right, but if you do your best to show me that you're trying really hard, that's good enough. I'm just giving them, 'In Mrs. F's classroom, what I want to see is just do your best.' So I think that just lines up with growth mindset."

Three teachers' statements were coded as growth mindset that did not fit into the previous two categories. When Teacher H was asked if she had encouraged a growth mindset into her own classroom, she stated, "I think I tried to just because I had to tell myself that I needed to kind of do it too." Teacher D mentioned that her students had been overheard doing "self-talks" when they were working their way through a task. The teacher gave this example when questioned about whether she felt her students were showing signs of taking on a growth mindset. Teacher F mentioned at the end of her interview that she had now connected growth mindset to some of the things she'd already said in class, and that she wanted to plan to talk more about growth mindset with her students. She stated:

> I was just casually talking about my expectation in math. Just do your best. I never connect that with ok this is a growth mindset. I will have to talk about growth mindset. That is something that I can do with my students tomorrow because we don't have tier tomorrow. Maybe that extra 30 minutes we can talk a little more about it.

4.1.2 Perceptions at Mid-year

All teachers were asked to rate their confidence in teaching number and operations by the end of their first semester of teaching. The first-year teachers were asked during their second interview in either December 2018 or January 2019. The second-year teachers were asked during their only interview in either September or October of 2018. Table 6 summarizes the teachers' numerical responses at the beginning and end of their first semester as well as their perceived reasons for any change in their confidence ratings. Four teachers' confidence increased by at least 0.5, and four teachers' confidence remained the same.

All teachers commented on better understanding of their students and/or curriculum. Teacher B referred to seeing progress in her students, and Teachers C, D, and H commented on knowing their students better and providing better instruction because of that knowledge. Teachers A and C mentioned being more at ease with the textbook while Teacher H commented on her comfort with supplementing textbook instruction. Only Teacher E based her confidence rating on the content being taught and her own understanding of that content.

Five teachers, Teachers A, B, D, F, and H, shared that they tried to implement growth mindset in their classrooms during the first semester. Although none reported using the specific term growth mindset, these teachers felt they were working to creating that classroom culture. These statements provide evidence of three teachers' attempts to implement growth mindset:

- Teacher A - “I tried to, but I think I could do better on that. I mean there are times when I hear kids say, ‘No, you just can’t do it yet.’ So I do hear some of them saying that. I feel like I hit on it more at the beginning of the year, and I haven’t as much now that we’re more set into things, but I try when I think of it.”
- Teacher B - “I feel like it’s something that I hope to encourage. I don’t know that I’ve created the culture that I want to yet. I’ve done some worrying on the culture of error and how to normalize mistakes, but I don’t know that I’ve done that super well. I don’t think that my kids feel any sort of shame from me, but I think still there’s some comparison among peers. In elementary school I was pretty high performing throughout school so I was pretty competitive so that’s not really growth mindset. That’s would be awesome if I could knock that out.”
- Teacher F - “I tried to. I’d say, ‘I’m going to stretch your brain.’ I’d tell them to use the resources on the walls, and I was letting them know adults are here to help. Our Second Step program talks about being assertive and asking for help, and they’re pretty vocal. I can picture one or two kids that are helpful and encouraging each other. One girl says ‘Mrs. O doesn’t want to hear I can’t do it.’ She tells them, ‘Just say you need help.’

Table 6. Teachers' confidence to teach number and operations

Teacher	Beginning of Year Confidence Rating	End of Semester Confidence Rating	Teachers' Perceptions of End of Semester Confidence
A	3.5 or 4	4	"I've gotten better at the pacing of the chapter."
B	3	3.5	"I think after doing it half a year, my kids actually do know stuff that they didn't before coming into the classroom so that's pretty exciting. I think that kind of helps because they actually do learn it when I teach it like this so that probably increases confidence because I can see progress for my kids."
C	1	3	"I started to understand kindergarten more. I understood my class, our dynamic, and academic levels. With the math content, I understood the flow of the Math In Focus manuals and the process of it. I got more confident because I didn't have to look at the book anymore. I could read through it and understand what were trying to teach."
D	3	4	"I'm feeling like I better understand what my kids are capable of and how much I can challenge them and what is going to be most engaging for them."
E	2 or 3	2 and 3	"I was still doing place value at that point so still a 2 and a 3. But once we got past place value, in February…So it kind of went up and down. I was a 2 then I was up to a 4 and a 5. Then I went back down to a 3."
F	4	4	"Umm, I'd say 4. I want to leave some room for improvement. I could say 5, but I'll say 4. I do still need to work on bringing some more hands-on stuff and games other than just the workbook in my instruction time."
G	4 or high 3	4	"I had already taught the same math curriculum and lessons at the same grade level in my internship the previous year. If I hadn't had that advantage, I wouldn't have been as confident. Confidence went up. Probably felt more confident later in the semester since I was more apt at running my own classroom."
H	1 or 2	3 or 4	"So I think once I realized that I didn't need to use the fractions from the textbook, because I hated it, and actually looked at different ways to do it and different ways to rewind and see where their foundational skills were from the previous year and then kind of go from that, I felt a lot better. I knew my students better at that point, and you kind of of ... I don't know. I just felt more comfortable teaching because math was one of my weaknesses in school. So that starts off my confidence low already. Once I kind of got in the groove, I was learning about different activities I could do with them and being able to work with smaller groups I just felt like I was benefiting them more."

4.2 Research Focus - Mathematical Content Knowledge

Teachers were asked to share perceptions of their preparation in the area of mathematical content knowledge. Teachers reported on their mathematics education experiences before and during university attendance, and some shared anecdotes of previous successes or failures in mathematics. They were asked for their perceptions about how their mathematics methods courses either did or did not improve their content knowledge to prepare them for teaching number and operations concepts during their first year of teaching. Teachers were also asked questions about the mathematics content they were most and least looking forward to teaching in the area of number and operations. They were then asked to describe their most memorable lesson by the end of the first semester of teaching. The data gathered in the area of mathematical content knowledge is discussed in the following two sections. First, perceptions of the two mathematics methods courses instruction in content are shared followed by beginning of year perceptions of content knowledge. Second, the end of semester perceptions of content knowledge and memories of lessons taught in the first semester are shared.

4.2.1 Perceptions at Beginning of the Year

The university attended by these participating teachers required completion of one mathematics methods course in the graduate program and two mathematics courses for undergraduates. One of the required undergraduate courses was the mathematics method course and one was a mathematics content course specifically geared towards elementary teachers, taught through the university's mathematics department. This section reports on the participating teachers' perceptions of the quality and quantity of content knowledge afforded in the course(s) they attended during their teacher preparation program. Because the purpose of the mathematics department course was the teaching of mathematics content, the teachers' perceptions of this course are discussed first. Then the teachers' perceptions about the methods course are shared.

The five teachers, Teachers A, B, C, G, and H, who completed the university's undergraduate teacher preparation program reported taking the content course through the math department. Although Teacher H believed she took the course, she couldn't remember any details about the course or its instructor. The memories of the other four teachers regarding the instructor seemed to indicate that they perceived their instructors to be knowledgeable in math, but may not have perceived them as useful in increasing their content knowledge. Teacher C said, “The teachers in that class, they were basically experts in math.” Teachers A and G reported difficulty understanding the professor. Teacher A stated, “We had a Chinese professor for that class. He was only there for a year.” Teacher G stated,

> Then the more content based one, it was a grad assistant who, bless his heart, we could barely understand him in the first place because he was from…I don’t even remember where from. He was just so smart. He wasn’t very patient with us. He was like, ‘Why aren’t you getting this? It’s just this, this, this, this…’ We were all just like, ok we don’t know what you are talking about. That poor guy. We were so frustrating because we just didn’t get it.

Teacher G shared another memory of the course and its instructor. She stated,

> I don’t remember the specific problem, but I remember we had a homework. We all came back the next day, and we all were like, ‘Did you know how to do this?’ Nobody. We were all clueless. We were like, we didn’t understand this at all. I remember we tried to ask him about it, and he just didn’t understand why we weren’t getting it because he just knows it. He was such a high level math guy. He just wasn’t able to explain it on our level which is sometimes how I feel with my third graders.

Three teachers’ perceptions seem to indicate concern that the professors from the math department were not able to help them become better mathematics teachers. For example, Teacher B stated, “But it was just kind of weird because the person’s not an educator at all. She’s only been like a college professor and researcher so when she’d try to give us educational strategies it was actually like, ‘I kind of learned the opposite of that in my college of ed, but thank you so…’” Teacher G felt, “It wasn’t like a teacher, like one of our education teachers. It was just from the math building so they weren’t taught how to teach math which is kind of a problem.” Teacher C reported:

> So it was kind of hard for me to understand because that teacher would explain it on our level, but then he kind of had a hard time explaining it to how we can explain it to younger kids. And I know he’s like really smart and very knowledgeable in math. So he

was just very intelligent and then for him to try to tell us how to explain the high level math, it was hard for me to basically convert whatever he's teaching us to a language that little kids can understand.

When asked about the specific concepts covered by the content course, the participants' memories revealed no similarities. Teacher G remembered lessons on different multiplication strategies, but no fractions. She initially referred to the course as "geometry for teachers" until she remembered other concepts taught. She stated, "Some of the geometry stuff was definitely, I think was higher than sixth grade. It was just content we might come across in any grade."

Teacher B reported that her content course "focused a lot on probability and number sequencing and things like that. Which I wasn't really sure why we did that." When asked what she meant by number sequencing, she replied, "Trying to remember. We talked a ton about a number set and then I think maybe we were trying to compare different numbers and then we had to put numbers in sets. We'd read problems and they'd describe like people's sandwiches and then we'd have to put their names in a set and later in the problem their names would have a numerical value and then that would be in the set… sorry…" She stopped trying to explain the concept the professor had attempted to teach.

Teachers A, B, and G shared memories of the structure of the class. Teacher A stated:

> I remember doing a lot of like workbook type stuff in that class. We did a lot of like literally an elementary workbook type thing. We worked in groups and then kind of discussed the answers. We came in and we sat in groups. We would work through some of the problems. We'd discuss how you would do them and then we would work through them. Then talk about them as a class.

In contrast, Teacher B's experience was described as sitting "in traditional rows and we had a textbook and it was taught out of that." Teacher B also shared:

> We got into fractions a little bit. I really mainly remember probability. I think for the last few weeks I'd read a book during it. So, sorry I don't really remember. It was just out of the book and we'd have weekly homework and the homework wasn't hard so I didn't think it was worth my time."

Teacher G remembered doing practice problems, and the grading practice of the course instructor. She reported, “It had a huge curve. It was such an interestingly done class that the curve - I had a 69 and it was a B. So it wasn’t the greatest.”

All participating teachers took the mathematics methods course, three as an online, graduate course and five experienced the undergraduate course live. Teachers shared memories and perceptions of their courses’ instructor, class structure and assignments, and the content instruction they experienced. Other than Teacher E whose online course was an eight week course, the teachers had a semester long methods course.

Six teachers shared what seemed to be positive memories about their instructor. For example, Teacher C stated her teachers were “very positive all the time.” Teacher F noted the challenge of getting the curriculum covered in one semester, ending with, “Yeah. It was pretty challenging, but she guided through really well.” Only Teachers G and H shared what might be considered concerns about their methods instructors. Teacher G stated, “The teacher, I think she was a grad assistant, too. She was in actual education…she was working on her masters I think, or maybe her doctorate in education and specifically in math. The teacher was much more lively.” Teacher H shared, “I’m pretty sure ours was like the guinea pig class.” When asked to explain she stated, “Because he was just new. I don’t think he’s ever taught that class before. No, I feel like he was kind of flying out of his butt the whole time.”

All the teachers commented on the collaboration, discussion, and group projects included in the structure of their methods courses. For all of the teachers except Teacher F, the collaboration and discussion were seen as routine experiences in their course. The following are the memories and perceptions shared by four teachers:

- Teacher G - “We worked a lot together. We were at tables in groups. It wasn’t like a traditional one in a row so we were in groups together. And anytime we had to do anything, like we made lesson plans, I think we made those with our block people because we were already paired up. When we looked at different apps, we had to pick an app and we looked at it together and presented it together.”
- Teacher E - “Well, I really liked the atmosphere of that class and the whole program because everyone was in constant contact with each other. We also had a lot of group

chats and group meetings where we would all conference call in and we could see each other and discuss our assignments and things like that. We had a message board where we could talk and share ideas. There were scenarios and it was how would you explain this? How would you tell them how to do it? We had a Facebook group. We made a private group where we could talk and get ideas from one another and share things."

- Teacher C - "A lot of our activities were always table talk or group activities, group projects. A lot of partners, especially in Block C."
- Teacher B - "We'd kind of do discussion that elementary students would have with discovering fractions and then we'd also have discussions then as teachers too. So as students and teachers and then the professor was like an active facilitator of that too. So like a lot of discussion and hands-on."

While Teacher F remembered some discussion and collaboration in her methods course, she also stated:

> Everything was online so, for a math course, we didn't have that much interaction going on with our group, with our cohort. I think there was one or two that we had to collaborate together, and come up with thinking about some math standards. We didn't have to create a lesson together, but more so discussing about pedagogy.

The teachers shared perceptions of assignments required in the methods courses. All the teachers were required to write one or two lesson plans, and all teachers except Teacher F were required to teach a lesson or two in their coordinating field placement that same semester. Teacher F also had a practicum that semester, but she was required to do observation assignments only. The seven teachers were required to videotape and submit their lesson to their instructors.

The three teachers who took the online, graduate level methods course, Teachers D, E, and F, commented positively about the feedback given on their lesson plans and videotaped lessons. Teachers E and D mentioned how useful it was to get feedback on their lessons from not only the professor, but also from their peers. Teacher E shared details about the use of message boards and conference calls where they could talk and share ideas. She stated, "There were scenarios and it was, how would you explain this? How would you tell them how to do it?"

These two teachers mentioned how much they learned from watching others teach different lessons. Teacher D stated:

> The videos were actually posted so I could watch other students, how they did a lesson and how they used a concept in their practicum which was kind of neat because everybody was at different grade levels and things like that. So watching a specific method being used in all of those different grade levels.

Other than the writing of mathematics lesson plans, the teachers' memories of methods course assignments varied. Four teachers, Teachers D, F, G, and H, remembered being assigned articles to read, but only one teacher, Teacher D, remembered having a required textbook. Teacher F reported:

> I had to just write it (lesson plans) up on a piece of paper and submit it to the professor because it's an online class. The assignment is for the whole week. The introductory lesson would happen on Monday. We would go and listen to the lecture that's posted online. Basically the assignment was out on Monday and we had to finish it by Sunday 12 o'clock. Everything was posted on Campus so you could go in there and there would be a whole week of assignments: the video, the assignment, the resources that can be with the assignment. You have to read all those articles if there were articles related to the assignment. Then produce our assignment and submit."

Four teachers, Teachers B, C, G, and H, were assigned the task of finding apps to utilize in mathematics lessons. Other than finding educational apps, Teacher H stated:

> Yeah. I honestly can't think of specific assignments because I felt like they were just kind of busy work. Like, I don't know, think about this word problem. How would you solve this problem? It was things like that.

Four of five undergraduate methods course participants, Teachers A, D, E, and F, remembered watching instructional videos of modeled lessons. Only one teacher remembered being assigned a student interview project. Teacher A had to create a short math task which she presented to a high, middle, and low ability student. She then had to interview each of those students to see how they were reasoning through the math problem.

Half of the teachers felt their methods course included discussions on mathematics assessments. Teachers B and C felt their instructor encouraged creative and informal assessment strategies to help them figure out what students were thinking and understanding. Teacher E felt she was taught to give assessments and analyze student work and data. Teacher G felt she probably discussed assessments in her class, but can't remember the details.

All teachers were asked whether they felt their mathematics methods course reviewed mathematical content during the course, and whether they felt the course increased their own mathematical content knowledge. When asked about the review of content, all teachers first commented on how they studied or reviewed the state or Common Core standards in the course. For example, Teacher A shared that they "talked a lot about the standards for math practice and how you should be focusing in on at least one of those during your lesson." Teacher F shared that they did vertical alignment activities with the standards to see the progression of a concept through the grades. Teachers B, C, and E shared similar memories about being given a specific skill or topic for which they had to identify the matching grade level and standard. For example, Teacher E remembered:

> There was a lot of assignments looking through those standards. Give us a math assignment, say quadrilaterals. Where does that fall under the Common Core? Now make a lesson based on where that falls under the Common Core.

The teachers were then asked specifically about the extent to which elementary mathematics content was reviewed in the methods course. All three participants who took the graduate level methods course said no content was reviewed. Teacher D said, "There was no refreshing your brain on how to do specific math." Teacher F explained, "Because it was Masters course she (the professor) was, the whole program kind of was assuming that this group of people already has some kind of mathematical background, or had some kind of college course regarding math." When asked if she understood the mathematics concepts better at the course's end, Teacher F replied, "Not really understanding the math any better, but understanding how to deliver math to the little kids better." Teacher E stated:

> Yeah, it didn't. I'm just going to chalk it up to we were the very first cohort. But it was nothing like, ok remember when you're teaching subtraction to do this…and remember when you're teaching multiplication … There was no refresher. It wasn't necessarily actual math. It was ways and how to teach it, I guess.

The five undergraduate program teachers who were asked about content review in their methods course shared a variety of experiences. Teachers A and B felt they increased their content knowledge in terms of conceptual understanding of computation algorithms. Teacher A shared, "I thought it was a pretty interesting class to break it to really make sure you're teaching the way that students are understanding the process, opposed to, understanding what's happening not just the process of where to put numbers." Teacher B mentioned learning the area model an another multiplication algorithm. She also stated:

> The things that stood out to me the most would be that road to the algorithm, so that by the end, when the students are doing it, they know why, and then like number sense so they know the value of the number, not just the symbol and how to manipulate it.

Teacher C didn't share memories about algorithm discussions, but simply referred to the professor sharing different strategies for how to explain concepts. Teachers G and H shared opposing memories of discussions about algorithms. Teacher G remembered discussing the traditional subtraction algorithm. She remembered discussing, "Their conceptualization of that – they just don't understand what they're doing. They don't understand the why. We talked about that a lot. Why is this this way and not just memorizing an algorithm for things." She did not remember discussing any multiplication, division, or fraction algorithms.

Teacher H did not remember discussing any specific algorithms. She shared that her professor would pose a problem and instruct them to solve it like a second grader, or like a fourth grader. She stated that the professor would then discuss common misconceptions of students solving those problems. When asked how discussing student misconceptions could have helped improve content knowledge, Teacher H answered:

> I guess when you first start, like my first year, you don't really understand what those misconceptions are because you're going in and you know how to add. You've never really seen a student's perspective and what other things they could end up doing, I guess.

Two of the three teachers who took the graduate level methods course mentioned class discussions or activities concerning algorithm use. For example, Teacher F described what she remembered about one of their assignments:

> Now you're teaching multiplication in fourth grade, but other than the traditional way, the olden way, how can you teach fourth grade friends to understand the concept of multiplication and have the deeper understanding of that?

Each teacher made reference to their courses' focus on mathematics concepts typically taught in middle and upper elementary grades. None of the teachers shared memories of lessons concerning concepts of number such as counting and cardinality typically introduced in kindergarten mathematics. For example, Teacher G stated, "We talked about number sense. I remember talking about it. We might have read an article about it, too. I can't remember a specific content that we talked about that would be a K or 1 specific." Teacher A shared multiple examples of activities in her methods course which focused on whole number computation algorithms and fraction concepts. She then shared details about the single lesson she remembered focusing on an early education mathematics concept. She said:

> We talked a lot about the equal sign like using the words 'same as' not 'equals' because sometimes then kids think that the equal sign is at the end of a problem whereas sometimes it might be 4 is the same as 2 + 2 it might not always be 2 + 2 equals 4. So a lot of like how we word things and how we show things I would say is a lot of what that class focused on. Just because, like I said, if we're just doing the process all the time like kids can see that but then they're not understanding the math that's happening.

Five teachers, including Teacher A, taught kindergarten or first grade during their first year. Teacher C also commented on the lack of focus on lower elementary mathematics:

> I know in my classes I felt like they were mostly first grade and up kind of math problems, and they don't really talk about kindergarten and how to explain numbers and

> number sense things. I think that's one part that they didn't go over in my math classes. I think it's the number sense. I don't remember much, but I know that it has helped me if I would have done like first grade to higher math because I don't remember going over things that we do in kindergarten. It's kind of hard to connect it to kindergarten because some of the things that we went over was just way too high for kindergarteners.

After discussing their teacher preparation memories, each teacher was asked to share opinions about which mathematics concepts in the area of number and operations they were most and least looking forward to teaching. By the end of the first semester, teachers were asked to reflect on these lessons. Then they were asked to describe and give details about their most memorable lesson in number and operations. The responses to this question are shared in the pedagogy section of this chapter because the teachers' stated rationale for choosing their best lesson were pedagogical in nature. None of the teachers chose their best lesson based on their content knowledge as was originally assumed when the interview questions were written. Table 7 provides the specific questions asked of each participant.

Five participating teachers taught kindergarten and first grade while three teachers taught third and fourth grade during their first year of teaching. Because the number and operations concepts and standards have different focal points at lower and upper elementary levels, the teachers were first sorted into two groups based on their teaching assignments - kindergarten to first grade and third to fourth grade. Then responses concerning the content they were most and least looking forward to teaching were analyzed for patterns.

It is interesting to note that there seemed to be difficulty identifying number and operations concepts when teachers were first asked, "What are you most looking forward to teaching in number and operations?" Teacher G asked whether multiplication was in that strand. Teacher D initially named teaching time, then mentioned teaching money. However, she then described how she would use money to teach addition and subtraction as an everyday life skill. Teachers C and E named teaching shapes as the concept they were looking forward to teaching although Teacher C proceeded to brainstorm how she could use the pattern blocks as counting practice for addition. For example, having the students sort the shapes, then count how many in each group. She mentioned having them count the number of sides on each shape as well.

Table 7. Interview questions concerning mathematics content knowledge

First Year Teacher Interview Questions		**Second Year Teacher Interview Questions**
First Interview	**Second Interview**	**Only Interview**
CONTENT KNOWLEDGE: 1. Thinking about your new classroom and students, what are you most looking forward to about teaching number and operations? Why? 2. What are you least looking forward to about teaching number and operations? Why?	CONTENT KNOWLEDGE: 1. After a semester of teaching, do you feel that your mathematics methods course provided enough content knowledge to effectively teach number and operations? 2. What were the strengths and weaknesses of the content knowledge in the mathematics methods course? 3. At your first interview, you said you were most looking forward to ____. How did the lessons go? (If needed prompt with: "Were the lessons what you imagined? Better? Worse?") 4. At your first interview, you said you were least looking forward to ___. How did those lessons go? (If needed prompt with: "Better? Worse? Did you plan anything special to prepare for those lessons?") 5. Looking back at this first semester, what is your memory of your best lesson? What made it so good and so memorable? 6. What would help you to teach more lessons like this lesson?	CONTENT KNOWLEDGE: 1. Before you started teaching your first year, what were you most looking forward to about teaching number and operations? Why? 2. Thinking back to when you had taught this lesson, how did the lessons go? (If needed prompt with: "Were the lessons what you imagined? Better? Worse?") 3. Before you started teaching your first year, what were you least looking forward to about teaching number and operations? Why? 4. Thinking back to when you had taught this lesson, how did the lessons go? (If needed prompt with: "Better? Worse? Did you plan anything special to prepare for those lessons?") 5. Thinking back to your first semester of teaching, what is your memory of your best lesson? What made it so good and so memorable? 6. What would help you to teach more lessons like this one?

Overall, Teachers A, C, D, E, and F, the five lower elementary teachers, were most looking forward to teaching concepts related to addition. Teacher A was looking forward to teaching addition and subtraction with larger numbers to 20 and 40. Teachers C and F were looking forward to building number sense leading into teaching addition. Teacher D planned to use money to teach addition because students "understand the concept of how money is used." Teacher E named multiple concepts to include addition. She stated, "I was really excited to teach about shapes, patterns, and addition and subtraction. Very basic addition and subtraction."

Two of the three upper elementary level teachers were most looking forward to teaching multiplication. The third upper elementary teacher was looking forward to teaching fractions. While the concepts were different, all three teachers stated that their own understanding of the content and pedagogy influenced their choices. Teacher G explained, "Maybe that's why I kind of like it (multiplication) because I just have it already there and I don't have to think about it a lot. I also like pulling in different ways to teach it." Teacher H said, "I just ever since school loved multiplication. I liked doing it different ways and getting larger. You know, multiplying 5 digits by 1 and finding those, and doing area model." Teacher B stated,

> I'm looking forward to fractions because this is their first, like this is the intro so I think it may be different than the subtraction with regrouping where they already know the algorithm. With fractions they don't yet so I can do a lot of discovery with it. And we spent a lot of time in our math methods with cuisenaire rods and fraction strips and things like that so I feel like I have tools to create discussion in the fractions.

For six teachers, the concept they were most looking forward to teaching were concepts they were confident in teaching due to their content and pedagogical knowledge for that concept. Only two teachers named challenging topics as the concept they were most looking forward to teaching. Teacher A stated, "That's probably my favorite kind of math is when you get into adding and subtracting a little bit bigger numbers, but it's also probably my area where I'll feel most challenged." Teacher D explained:

> It was really scary to teach place value, number bonds, and things I wasn't familiar with teaching. But for me I had to completely lay that foundation for them and that was

> extremely intimidating and scary for me because if I messed up they would be in trouble. And I did mess up, you know, and it's all about learning.

When asked about the concept they were least looking forward to teaching, the lower elementary teachers' responses split three ways. Teacher F stated that making her students memorize facts was something she wasn't looking forward to. Teacher C named concepts related to number sense and stated:

> They come to you and they might not even know their number 0 to 10. Some don't even know what zero is or that zero is a number. I was not looking forward to going back to the basics. I was kind of nervous because how am I going to teach math and they don't even know what 1 looks like or if they can write their numbers.

For three teachers, the level of difficulty for the student seemed to influence their choice of concepts they were least looking forward to teaching. Teachers A, D, and E's responses seemed to be related to the wording associated with mathematics concepts. Teacher A didn't name a particular number and operation concept, but worried in general about her ability to explain the concepts to students especially if they were struggling. She stated, "Cause sometimes it's so simple in your head and you don't know how else to explain it." Teacher D was worried about word problems with her first graders. "When we've gotten done a unit on addition and a unit on subtraction and putting the together and kind of distinguishing when we use addition and when we use subtraction." Teacher E was least looking forward to teaching place value because of the wording of questions. She stated:

> I knew the kids were going to have a hard time with it just by the wording. You know, how much is 2 tens? That's really confusing to first graders. How many tens are in 52, the number 52? Those exact words. That is really intimidating.

The three upper level elementary teachers named different concepts they were least looking forward to teaching although for similar reasons. Teacher B named multiplication, Teacher G named measurement, and Teacher H named fractions. Each explained that these concepts were weaknesses of their own although for Teacher B the weakness was in pedagogy not content knowledge. Teacher B explained, "To really facilitate good discussions with introducing multiplication and division, I don't feel like I have a ton of skills on that." Teacher G named measurement as a "struggle," and Teacher H described fraction concerns by saying, "It was just one of my weaknesses growing up and we didn't really touch base on it in blocks, and I didn't get to see it during interning." For these three teachers, the level of difficulty for the teacher seemed to influence their choice of concepts they were least looking forward to teaching.

4.2.2 Perceptions at Mid-year

At the end of the first semester of teaching, the first year teachers were asked to reflect on the lessons they had been most and least looking forward to teaching in number and operations. Teachers were asked to share what preparation and planning they did to prepare for the lessons, and to report their perceptions on how the lessons went. The four second year teachers were asked to share responses to these same questions during their single interview.

Three teachers, Teachers A, B, and D, had not yet taught the concept they were most looking forward to teaching. Teacher A reported she hasn't thought about the lessons yet, but Teachers B and D reported having materials in mind for the lessons. Teacher F was least looking forward to having students memorize basic facts which she now felt was more for first and second grade students. She reported that she thinks she won't have to do this with her students because they will learn a lot of their facts through repetition in the activities she has planned.

Teachers F and G felt the lessons they were most looking forward to teaching went well, and Teachers A and D felt the lessons they were least looking forward to teaching went well. Teacher F reported that she didn't really follow the Math in Focus text, but used ten frames and magnet chips on the board to model place value and number sense. Teacher D, who was not looking forward to word problems with first grade, created anchor charts and showed the students how to draw pictures or act out the problems with manipulatives. She noted,

> It wasn't terrible. The reading part was a struggle, but I think that just solving the word problems wasn't as bad as I had anticipated. Giving them context was really helpful, and I had a lot of fun making silly stories and word problems up. It helped engage them a little bit more than an equation on a piece of paper.

Teachers B, G, and H shared that the lesson they were least looking forward to went well. Each teacher researched lesson and pedagogy ideas by looking at supplemental materials and conferencing with other grade level teachers. None of the teachers used the district curriculum or materials from their methods courses for teaching these lessons. Teacher H stated,

> I think it went better than I thought because I was like, this textbook is not going to work, so let me rewind. I really went back further. I went through third grade standards and looked at those. Talked to the other third grade teachers and asked, hey what have you done? Where do I need to start? Just so I can see where they need to be and where we need to go. Once I knew that, I was able to really go in and find different strategies.

Teacher H was the only teacher to report that the lesson she was most looking forward to teaching did not go well. She shared, “At that point, I was still trying to use Math in Focus instead of going off kind of on my own. I was trying to follow what it was doing and it just wasn’t working for my kids.” While Teacher E used terms like “scary” and “intimidating” to describe both the lessons she was most and least looking forward to teaching, she reported that the lessons were “fine”. She also felt that her students took much longer to learn the concepts than they should have because the district curriculum seemed confusing.

Teachers were asked whether they felt their mathematics methods course provided content knowledge preparation for teaching elementary mathematics concepts in number and operations. The three teachers who took the graduate level course said it did not. Teacher D stated, “I think I learned more in the classroom and from colleagues and other teachers.” Teacher F stated, “No. This was a grade course so the teacher didn’t lecture or teach. Students created items and shared them with the teacher not with each other.” Teacher E commented on the short length of her eight week course as a reason why there wasn’t content review integrated into the methods course.

Teachers A, B, and G felt there was more content knowledge reviewed, but that it was only in certain topics. For example, Teacher B stated, “I think in terms of understanding concepts, we really dove in for fractions so that I’ll be prepared for. With the multiplication, I don’t know. I had a strong understanding, but I don’t know it was from my methods course.”

Teacher G stated, "Some of it. Like the rules that expire one helped because I remember thinking when I learned negative numbers that I could subtract seven from four." As a third grade teacher Teacher G wanted to make sure she explained mathematics rules correctly so that it wouldn't "mess up how they're thinking" in the future. Therefore, she didn't want to tell them they can't subtract a bigger number from a smaller number because they will do that when they start to work with negative numbers. Teachers C and H did not feel the course provided content knowledge review or reteaching.

4.3 Research Focus - Mathematical Pedagogical Knowledge

Teachers were asked about their constructivist pedagogical knowledge for teaching mathematics, and how their mathematics methods coursework did or did not introduce, model, or encourage those pedagogies. Teachers were asked for their perceptions of how constructivist pedagogies may or may not have helped prepare them for teaching number and operations concepts in their first year of teaching. Teachers were asked about their own implementation of those pedagogies. Had they already implemented those pedagogies in their classroom, or did they plan to implement them during their first year of teaching elementary mathematics?

Teachers were also asked to describe a typical mathematics lesson in their classroom at the start and end of their first semester of teaching. Teachers were asked to give their opinions as to the cause of any changes that might have occurred in the structure and pedagogy of their typical mathematics lessons by the end of their first semester of teaching.

The teachers were then asked to share advice they might have for mathematics methods course instructors which could prepare teachers to effectively teach number and operations concepts. Table 8, on the following page, shows the interview questions concerning mathematical pedagogical knowledge and advice mathematics methods course instructors..

The data gathered from these questions is shared in the following two sections. First, the memories of constructivist pedagogies introduced or modeled in their mathematics methods course is shared. Second, the teachers' own implementation of constructivist pedagogies and descriptions of typical lessons at the start and end of their first semester of teaching is shared. The perceived strengths and weaknesses of their methods courses, and the teachers' advice to methods course instructors is shared last.

Table 8. Interview questions concerning mathematical pedagogy

First Year Teacher Interview Questions		**Second Year Teacher Interview Questions**
First Interview	**Second Interview**	**Only Interview**
PEDAGOGY: 1. Describe a typical mathematics lesson that you've taught up to this point.	PEDAGOGY: 1. Describe a typical mathematics lesson that you've taught up to this point. What changes, if any, have you made to your math lessons since the start of the year? Why? 2. After a semester of teaching, which philosophies and strategies taught in your mathematics methods course match up to what you experienced in your first classroom? 3. What advice would you give instructors of mathematics methods courses that could help ensure teachers are ready to effectively teach number and operations concepts?	PEDAGOGY: 1. Describe a typical mathematics lesson at the start of your first semester teaching. 2. Describe a typical mathematics lesson by the end of the first semester. What changes, if any, have you made to your math lessons since the start of the year? Why? 3. Thinking back to your mathematics methods course, what strategies, resources, or lessons did you utilized in your lessons the first semester of teaching? 4. What advice would you give instructors of mathematics methods courses that could help ensure teachers are ready to effectively teach number and operations concepts?

4.3.1 Perceptions at Beginning of the Year

The teachers were asked to share their perceptions of how their mathematics methods course introduced and modeled constructivist pedagogies, and to what extent they felt the pedagogies affected their readiness to teach number and operation concepts. While there was some initial confusion about constructivism, the teachers' beginning of the year responses seemed to suggest the methods courses positively affected their pedagogical knowledge.

When first asked about constructivist pedagogies used in their mathematics methods course, none of the teachers recognized the term constructivist. Teacher D asked, "Like constructive criticism? Things like that?" Teacher E replied, " Hmmm. That's a hard question." Teacher G stated, "It sounds familiar, but I'm going to be honest. I don't really quite remember what that is." Teacher B also seemed to misunderstand the term. She stated, "I think if we were given writing assignments or had to create lesson plans, I think constructivism was a really common strategy that was listed."

Despite the initial uncertainty of the term, the teachers all shared examples of constructivist pedagogies introduced and encouraged in their mathematics methods course. All teachers gave examples of remembered activities that used open-ended tasks, inquiry-based lessons, and hands-on manipulatives. However, the frequency of usage and the specific tasks and manipulatives experienced varied between teachers.

Teachers A, B, C, D, and H shared memories of solving word problems or mathematics tasks in their methods course. While Teacher B was the only teacher to specifically use the term inquiry when describing her experiences, six teachers remembered their professor encouraging the use of problems that allowed for multiple strategies and/or multiple answers. Teacher A stated, "Sometimes it would be such an open-ended question that there was no right answer." Teacher B remembered talking "a lot about the importance of problems that have multiple strategies, multiple answers and then also missing middle, missing beginning, and missing end." Teacher D learned that "kids could have lots of different answers as opposed to looking for one answer," and Teacher D remembered talking about problem solving strategies during the course. Teacher E remembered being told to teach children "to understand there are many different ways to solve a problem and how to solve it."

All the teachers specifically mentioned their methods course encouraging them to see the importance of students' conceptual understanding as opposed to procedural understanding of the mathematics they were teaching. The following responses are examples of what three teachers learned in their methods course:

- Teacher A - "So most of our like lesson structures, we talked about introducing it with manipulatives moving to picture model then more abstract understanding and that was I would say the basic growth pattern we talked about."
- Teacher D - "We did charts when we were learning 1,000s and 10,000s. There was a lot more explaining. We want the kids to understand why it is this way, what it means, why are we carrying the one. What happens when we're carrying the one? Which I appreciated because again, when I was in school, it didn't matter why we did it, it was just what we did. It was like I don't know she told me to carry the one. That's what I'm going to do. So I think there's a lot more focus on making sure and asking your students as you're watching them do it… Why did you just do it? What does that mean?"
- Teacher F - "It was a lot about trying the make the kids understand the mathematical concepts not giving children worksheet after worksheet and having them memorize what they've learned. We went over a lot of strategies and teaching techniques and the materials we can use to help children to understand the concept."

Seven teachers made statements that indicate their methods course stressed the importance of having students explain their thinking and their answers to problems. Teachers A, B, and C had professors who made them explain their thinking as they did mathematics tasks in the methods classroom. Teacher C's professor also explained how students' verbal explanations could be used as an alternate form of assessment especially in the younger grades.

Teachers D, E, F, and H remembered being taught to require students to explain their answers. Teacher D watched an instructional video modeling number talks. Teacher F was taught to ask students "high level questions to get the thinking going." Teacher H stated that her professor also modeled redirecting students when necessary if they gave wrong answers to a math task during their explanations.

Seven teachers were taught the importance of group work and discussions in their mathematics methods course. Teachers A, B, and H experienced group discussions during each methods class session. Teacher C remembered being taught how to facilitate “table talks” in her mathematics lessons to encourage everyone to talk during group work. Teachers D and F were encouraged to use centers in their mathematics lessons. Teacher F also remembered an assignment in her methods course to study differentiation, and how using small groups can help differentiate and support students at different levels of learning. Teacher E made multiple comments about the importance of group work:

> Making sure that the kids are in their groups and they're working together and they're learning from each other. The math class just mainly focused a lot on cooperative learning. It really hit Kagan cooperative learning very hard. We learned it was important to do group work, not always individual work so the kids can learn from each other and with each other. If there is maybe a student that's higher in math and a student that's lower math, if they're in the same group, they can teach each other and help each other.

Teachers A, D, and G, mentioned scaffolding to build knowledge as a constructivist pedagogy introduced and encouraged by their methods professor. These examples are evidence of two teachers' learning:

- Teacher D - “Before I could teach them addition we're doing number bonds and making that connection and building those strong foundations and understanding. They have to have number sense first. You have to connect previous knowledge to be able to engage the part of the brain that's going to learn new knowledge.”
- Teacher G - “They're building off of background knowledge. I can specifically think of subtracting. They have to start with bigger numbers on top so they're not regrouping at all. Otherwise, if you go straight into subtracting and there's a zero up there, they don't know what to do.”

Hands-on manipulative use was encouraged by all of the teachers' methods courses although there were differences in the types of manipulatives remembered, the methods for introducing the manipulatives, and the frequency of manipulative usage in the courses. Table 9 summarizes interview responses of teachers who remembered methods course lessons using specific types of manipulatives.

Table 9. Manipulatives remembered from mathematics methods course experiences

Teacher:	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**
Base 10 Blocks		X				X		
Counters							X	
Tangrams	X						X	
Cuisenaire Rods	X	X						
Geometric Pattern Blocks								X
Fraction Strips	X	X						
Unifix Cubes	X			X	X			
Bucket Scales				X				

While the specific manipulatives named by teachers varied, all mentioned being encouraged to find and use manipulatives in their lessons. Teacher D stated, "What I learned was manipulatives. Using different manipulatives because lots of children learn many different ways, as we know." When asked if the professor introduced her to manipulatives, Teacher F remembered, "Not really introduced to us, but more of, ok there are manipulatives that you can use. What are those? Let's find out." Teacher D stated that while her professor never modeled manipulatives directly, they were assigned instructional videos to watch. Teacher D specifically remembered watching a video of a modeled lesson which used unifix cubes and bucket scales. Teacher H also shared that the professor did not use manipulatives in class. However, Teacher H stated, "It was more when we talked about the primary grade levels is the only time they talked about manipulatives."

Three teachers mentioned that they had experiences with manipulatives during the field placement they completed in conjunction with the methods course, but not during the methods course sessions. Teacher D shared, "I didn't physically have access through my class to those things. It was something that when I was doing my practicum, I would have to ask if I could use those materials." Teacher D reported that she was able to observe and use tangrams, cuisenaire rods, base 10 blocks, fraction strips, and counters during that field placement. Teacher H reported teaching a lesson with base 10 blocks for her practicum. Teacher C reported:

> There was not (a lot of manipulatives). I know that there was a project that we had to do, like how to use manipulatives and how to incorporate it into your lesson, but the school didn't actually provide manipulatives. We had to go into the schools and borrow from our teacher that we were working with to use those manipulatives.

Teachers A, C, D, and H remembered methods course lessons on drawing or using visual models to help students understand mathematics concepts or solve mathematics problems. Teacher A remembered her professor stressing that pictorial representations were needed after manipulative use. Teacher C mentioned that drawing it out was another strategy she could use with students to help them understand. Teacher D mentioned bar diagrams that students in upper elementary classrooms can use as models instead of manipulatives which can help them determine the necessary function for a word problem solution, or to compare values of fractions. Teacher H remembered her professor drawing a place value chart on the board to model place value and the steps for addition and subtraction of whole numbers.

Six teachers mentioned their methods course including the use of games to teach mathematics in some way. Teachers B, C, G, and H had assignments to find games or apps to use in mathematics lessons. For example, Teacher B found an app utilizing base 10 blocks and Teacher G was shown an app that utilized tangrams. Teachers D and E mentioned that they got to observe and experience many mathematics games in their field placement for the methods course. Only Teacher G remembered details of two mathematics games played during the methods course class sessions.

Some teachers' responses about pedagogy don't fit into the previously mentioned categories, but do show evidence of constructivist pedagogies introduced or modeled in the teachers' mathematics methods courses. These memories include sequencing, increasing depth of knowledge levels of tasks, and understanding different types of learners.

Teacher B shared memories of her methods course professor practicing and modeling sequencing. While the preservice teachers worked on a given mathematics task, the professor would walk around the room and take photographs of solutions. These solutions were specifically chosen as a sample of the different methods the teachers used to solve the problem. The professor then showed them to the whole class in a specific sequence, from the most basic or simplest solution to the most abstract or advanced solution. This modeled for the preservice teachers the correct sequence in which they should share their own students' solutions to tasks.

Teacher B also shared a memory of an assignment where they were asked to take a task from a mathematics curriculum and adapt it to have a higher depth of knowledge, or DOK, level. Teacher B felt like this was a useful lesson which also allowed them to study mathematics curriculum layouts.

Teachers C and E felt their methods course instructor stressed the different types of learning styles although there seemed to be some confusion about the correct terminology. For example, Teacher E stated, "Really focusing on the different types of ways that kids learn, kinesthetic, oral and visual." Teacher C stated:

> I know there are different kinds of learners. There are visual learners, movement learners, kinesthetic learners. I personally liked that kind of learning and teaching strategy. I just used whatever my kids can understand most.

None of the teachers shared methods course memories that specifically addressed culturally sustaining pedagogy or mathematical modeling as described in the GAIMME report. Teacher E did remember talking about being responsive to student interests, and Teachers A, C, and G remember being taught to be responsive to student ability levels and learning styles. However, none of the teachers referenced students' cultures as consideration in determining pedagogy. None of the teachers specifically referenced mathematical modeling tasks although problem solving was a part of many of the teachers' methods experiences.

After the teachers shared their memories of pedagogies introduced and modeled in their methods course, they were asked which pedagogies they had already implemented by the time of their first interview. Scaffolding, inquiry lessons, requiring students to explain their answers, and the use of mathematics games and apps were the pedagogies mentioned the least. Only Teacher A specifically mentioned the scaffolding of her lessons when she noted that some students were having difficulty transitioning from counting on their fingers to using number lines and writing equations. She mentioned the need to go back and make those connections. She reported that she was "making sure to really break things down into smaller chunks." Only Teacher B mentioned using inquiry-based lessons or mathematics tasks with her class on a regular basis.

Teachers A, D, and H stated that they require students to explain their thinking and answers although Teacher A explained she didn't always require this. She stated,

> During whole group, honestly not as much. Small group, yes. When I'm working with a smaller group of students then I think it's easier to take the time to talk more one on one with the student. With whole group it can be a lot because when they're finished and we're not quite ready to move on then they get off task. Then it's more challenging to stop and talk about those things.

Teacher H reported that she asked students to explain their thinking when they get frustrated so she could see where they were confused. Then she used redirecting techniques from her methods professor to give hints which could lead the student towards the correct answer. Teacher D has a rule in her classroom called, "Two before me." If she is working with a group and someone needs help, they are to ask two peers for help before interrupting the teacher. Teacher D also used a lot of "Think-pair-share" activities because:

> I'm telling them to turn to a partner and tell them their answer. That's an instant activity because if you disagree, they have to convince each other that they have the right answer. That's really cool because they literally have to explain it to each other and come up with a consensus.

Teachers B, D, and G reported that they had already used mathematics games or apps in their own class which they learned of in their methods courses. Teacher B used an app for digital base 10 blocks. Teachers D and G had used games and apps from their coursework. Teacher D used games and apps to practice addition skills. Teacher G used a game called, "Close to 100" with her third graders, and she attempted to use an app called "Slide Math" although the school district's firewall wouldn't allow the game to work.

The use of manipulatives, group work or centers, and the use of diagrams and models were the methods course pedagogies mentioned most often by the participating teachers. All teachers named manipulatives that they had already utilized in their beginning of the year mathematics lessons to varying degrees. Table 10 summarizes specific manipulatives participating teachers named when asked about manipulatives used by the first interview. Five teachers had already utilized group work or centers in their classrooms, and five teachers had utilized visual representations in their lessons.

Table 10. Specific manipulatives used by first teacher interview

Teacher:	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**
Base 10 Blocks		X					X	X
Counters	X		X		X	X		
Tangrams								
Cuisenaire Rods								
Geometric Pattern Blocks				X				
Fraction Strips								
Unifix Cubes				X		X		
Bucket Scales								
Geoboards				X				

Connecting or unifix cubes, counters, base 10 blocks, geometric pattern blocks, and geoboards were specifically named by teachers when asked which manipulatives they had already used in their classrooms at the beginning of the year. Teacher C also stated she had plans for use of Rekenrek bead with her students. The following teachers' statements are memories of introducing manipulatives to students.

- Teacher A - "Sometimes the hands-on especially first grade is chaotic because they start playing with the things or… but I keep on reminding myself that is, in my opinion, the best way for them to start out because then they're physically moving them and they can count them and they can touch them. So taking the time to really do the hands-on even though that is sometimes time consuming and that can be chaotic and slow."
- When asked about managing the use of base 10 blocks up through the 10,000 place value, Teacher B stated, "That's why I started using the app because we have those 1,000 cubes

and they’re a mess. They get stacked up when they’re trying to do 8,000, so that’s why we moved to the app on that one.”

- Teacher D - “My first couple of days of school I was doing just manipulative centers. So they were using the geoboards. They were making patterns with the unifix cubes. They were making pictures with the geometric shapes. Things like that, and getting kind of familiar with those manipulatives.”
- Teacher F - “The beginning of the school year, when we didn’t do Math in Focus, I was printing off from Teachers Pay Teachers and having them work on those. Taking out the math tubs and having them explore what’s in the math tubs. I’m using the unifix cubes and counting chips. I have not done the base 10 blocks yet.”

While the use of manipulatives was mentioned by every teacher, the frequency of use and the specific manipulatives used differed between teachers by the time of the first interview. For example, all teachers mentioned the use of base 10, cubes, or counters although Teacher F didn’t use them very often. She stated,

> In my class, since I’m doing kinder level, right now I don’t think I can really dive into giving them a lot of hands-on materials yet. It’s more I have 30 minutes core time, then we have after that 30 minutes of tier then 15 minutes of another chunk of core time. I’m trying to figure out how I can squeeze in the hands-on activities that they can do. I’m trying to do as much as possible if it’s in that core content that we introduce in that book, like in the pacing guide the teacher’s guide, but not the additional stuff.

In contrast, Teacher D had already used geoboards, unifix cubes, geometric pattern blocks, and number lines. Teachers A and C had already used counters. The third and fourth grade teachers, Teachers B, G, and H, all mentioned having used base 10 blocks.

Five teachers, Teachers A, C, D, E, and F, had used group work or centers by the time of the first interview. For example, Teacher A shared, “They loved it. They did it for probably 20 minutes the first day. They had time to build number bonds and challenge partners to create new number bonds.” Teacher F used some small group instruction and centers in the first weeks of school, but then switched to more whole group when scheduling and pacing issues arose. With less core mathematics instruction time, she reported that she felt the need to stick to the textbook curriculum to stay on pace with the rest of her grade level. Teacher D stated:

> If I have my kids sitting on the carpet, looking at me, and I'm talking and they're quiet, their attention span is very short. It's going to work for a very short amount of time. If I can quickly explain some centers and get them out there working, we get a lot more done. We're doing a lot of centers.

Teacher E reported that her experiences with the use of centers had both positive and negative consequences. Her comments seemed to indicate that she was encouraged to use centers during her methods course and student teaching. However, it appears she did not fully understand how to monitor her students' progress during center implementation. These quotes show her opposing perceptions of center use:

- "I did lots of hands-on stuff. Lots of individualized stuff. I would call back really small groups. I would have white boards, and I would have them practice their addition that way and look at their number lines that way. I would have counting bears and counting blocks and cubes, just tons and tons of stuff."
- "I'm going to pull you six back here with me, set up the dividers, and I'm going to give you the test. Then when one person gets done we're going to switch out. So if one kid sits there the whole time, and they don't finish, ok we'll just finish tomorrow. It's not that big of a deal. You're not going to fail because you didn't finish today."
- "So when I started school, right away, so we're learning shapes. So I'm just going to have us do centers every day. Every single time we do math, it's going to be centers. Shapes worksheet, shapes game, shapes to play with. Every day. I did that, and that's where I really am regretful. Later on I realized kids didn't really know things because I didn't really know how to teach math, I guess. I wasn't taking the time to see what kids knew what. What kids had learned what, and it was more like I wasn't giving them that individual help. I wasn't doing the informal observations where you say that, oh Sally is learning because she's doing that. It was more so just walking around making the kids were good and doing their centers. I legitimately didn't know, honestly because I didn't really know what I was doing. It was more behavior management. Making sure that they

were quiet and they were doing their work because that's where I came from, you know, two months earlier?"

By the first interview, Teachers A, B, C, D, and E, had used models or diagrams during mathematics lessons. Teachers A and E had used number bond drawings. Teacher B shared that she required students to first model numbers with base 10 blocks, the draw pictures of those models. Teacher C asked students to draw pictures to represent numbers in her kindergarten class. Teachers A and D also referred to students drawing pictures. Teachers D and E referred to the use of number line drawings, and Teacher D used place value charts to model numbers.

During the first interview, teachers were asked to describe a typical mathematics lesson in terms of structure and pedagogy. All of the teachers reported following the district curriculum, Math in Focus, by using a combination of the teacher's manual, online components to the curriculum, textbooks, and workbook pages. All of the teachers reported using supplementing materials at some point in their beginning of year lessons as well. The following teacher statements are examples of evidence gathered during the first interview that seemed to show a reliance on the district curriculum for their instruction:

- Teacher C - "I'm still learning how to use some of the Math in Focus and some of the resources, but most of the resources I use are Teachers Pay Teachers and trying to find more creative ways because I don't like using workbooks all the time."
- Teacher D - "I feel like we're given a curriculum - like, oh, we're using Math in Focus. That's the curriculum that we're using and it's beat into our head that we have to have fidelity to that particular curriculum. So as a first year teacher and even as a grad student, I'm like this is exactly what I need to do. I need to do, I'm on Chapter 1, Lesson 2 on this day and this is what we're going to do. Also, trying to keep up with the pacing guide is very hard because I feel like my kids...They want me to spend two days on a number line and I need a week on number line. So feeling like I'm covering all the content and at the same time adapting it so it fits the needs of my students - that's my biggest struggle."
- Teacher E - "I thought I need to go by what Math in Focus is telling me. I need to do some whole group. I need to do the worksheets they provide and move on."

- Teacher F - "I'll be using my teacher's guided book and teaching the concept. Then manipulatives. Doing a little bit of manipulatives and games that's in the teacher's guide book. I'm trying to do as much as possible if it's in that core content that we introduce in that book like in the pacing guide the teacher's guide, but not the additional stuff. As a first year teacher I feel like that I have to (follow the guide). I try. I try. I know as I get experience, if I know the content well enough, then I can plan to tweak. But right now because I'm not really sure what's coming up next, I try to follow what's there so it will be smooth. I love Teacher Pay Teacher, but so far I'm just focusing on Math in Focus and trying to find my pace. Trying to find my children's pace, and I want to sync with what my team are doing too. So, so far I'm just focusing on Math in Focus."
- Teacher G - "I stuck to the manual a lot because I wasn't quite sure what to, exactly on my own… I had watched my cooperating teacher, but she's taught for so long and she just has it down, and she doesn't really need to look at the manual and she knows just what to do. So I kind of kept the manual right in front of me so I knew exactly what I was supposed to be doing that day."

Seven teachers reported they typically started the lesson as whole group instruction. Teacher H described her lesson as a "I do, We do, then You do" structure which seemed to match most teachers' descriptions. Teacher B was the only teacher whose lesson structure seemed to differ. She shared, "So I tried to keep it a variety per week I think I do a lot of pairing up with different strategies, sharing and comparing usually and then when I have the whole class, so I also try do a lot of literature integration." Teacher A also commented on trying different approaches to introducing concepts. Teacher A stated:

> I'd say yeah typical lesson, do something hands-on in the beginning, or do something to get them thinking or kind of review in the beginning and then in the middle there's usually whole group. I try to mix it up though because I know I have a lot of levels and so trying to keep those higher kids challenged and engaged but also not letting the lower ones, you know get all confused and behind.

Seven teachers referred to using workbook pages, worksheets, or "individual work" as a typical component of a daily mathematics lesson. Teachers A, C, and F specifically mentioned using the curriculum's workbook pages on a regular basis. Teachers D and E used centers in which a worksheet was one of the activities. Teachers G and H referred to independent seat work, but did not specific if it was the curriculum workbook.

Six of the teachers shared examples of how they monitor their students' progress during the lessons using informal assessment strategies. Teachers A and H had students drawing solutions on their desks or white boards. Teacher A would also place a star at the bottom of the workbook page if the student got the answers correct. Teachers B and D used class discussions and whole group reflections at the end of their lessons to review what they'd learned. The statements below from Teachers F and G show how they use individual check in strategies during independent work to monitor student progress.

- Teacher F - "I just walk around and see how they're doing. Stop at certain friends who need more of my attention and start reteaching and counting with them. Showing them how to write. When they're done with their workbook pages then I check. The fast kids, if they're done earlier than other kids I have some extra sheets."
- Teacher G - "At the end of it (the lesson), we'd be either playing it in a game or working on something individually so I could go around and see what they were doing and if they got it or not that day."

All of the teachers refer to discussions taking place in their math lessons as part of the whole class portion of the lesson. Only Teacher B referred to having students explain their strategies and processes for getting a particular answer to a problem. Teacher G referred to group or partner work, and Teacher A referred to partner work where they could challenge each other to build number bonds, for example. Teacher D encouraged students to ask each other for help with the "Two before me" rule in her classroom. Teacher E had students work together in groups to complete the station activities. Teacher F noted that she had seen children helping each other count to complete activities at their tables and partner work for building numbers with cubes. Teachers C and H did not refer to group or partner discussions as a regular part of their mathematics lessons at the start of the year.

Only five teachers specifically mentioned the use of hands-on manipulatives as a part of a typical mathematics lesson at the start of the year. The following statements show examples of how manipulatives were implemented in typical, beginning of year mathematics lessons:

- Teacher A - "At the beginning we do a lot of hands-on then it moves more to just numbers or just our own pictures instead of manipulatives."
- Teacher C - "Then I'd move on to the physically seeing like what a 6 might look like. I used the cubes a lot. I even had pom-poms in the front of my classroom. I'd just tell a friend count out 6 blue pom-poms and then we counted all together. Then we count using our fingers."
- Teacher F - "Doing a little bit of manipulatives and games that's in the teacher's guide book. I will do, pretty much right because it's the number recognition, I use the snapping cubes, the unifix cubes for them to show how many and they can identify this is 5 cubes."

4.3.2 Perceptions at Mid-year

Teachers were also asked to describe their typical mathematics lesson by December of their first semester of teaching. Teachers were asked to share their perceptions regarding the causes to any changes they may have made to their typical lesson. All the teachers reported using the same structure of "I do, We do, You do" with whole group instruction usually coming first. The changes that teachers made to their typical lessons were caused by supplementing activity ideas from outside of the district curriculum, changes in the pacing of the curriculum, and changes in the use of small group instruction.

All the teachers except for Teachers A and F reported using more supplemental activities in their typical lessons by the end of their first semester. Teachers C, D, E, and G made more use of mathematics games in their lessons. Teachers C, D, and E used manipulatives more regularly to supplement the curriculum workbook pages. Teacher C noted the students were faster and more independent using the manipulatives by the end of the semester especially after the expectations had been set for their usage. Teacher H used ideas from Pinterest and Teachers Pay Teachers to supplement her textbook lessons. Table 11 summarizes specific manipulatives each teacher named when asked about manipulative use by the end of the first semester.

Table 11. Specific manipulatives used by second teacher interview

Teacher:	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**
Base 10 Blocks	X	X		X		X	X	X
Counters	X		X		X	X		
Tangrams								
Cuisenaire Rods							X	
Geometric Pattern Blocks	X		X	X				
Fraction Strips								
Unifix Cubes			X	X	X	X	X	
Bucket Scales								
Geoboards				X				
Play Money	X						X	

Teachers E and G specifically mentioned changes to their typical lesson structure that resulted from changes they made in the pacing of their curriculum. Teacher E stated, "It became more focused on making sure they understood one thing before jumping to the next thing. Whereas at the beginning, it was we need to know it all right now." Teacher G made a similar statement when she said, "That got better just being more comfortable stopping somewhere and reviewing where we needed to. Before, I felt like I needed to stick to ok, we do this this week and this next week, and I need to keep going."

Teachers D, E, F, G, and H reported that their typical lessons changed to include more stations, centers, and small group instruction by the end of the semester. Teacher D noted that her students love the centers and get upset if she doesn't implement them regularly. She stated that she usually incorporates centers after she's taught the concepts and expectations for each center, but has rarely used centers to introduce new concepts. Teacher E reported that her use of stations "changed how I interacted with the kinds and the structure of the classroom. Pulling them back where I can help them." Teacher H used one of her stations as a spiraling activity to review previously taught skills. Teachers A and F had begun enrichment worksheets or activities for “early finishers.” Teacher F shared the picture below of her game station shelf which she started to implement at the end of the semester.

Figure 3. Photo of Teacher F’s mathematics games for use during centers.

Each teacher was asked to share the memory of their best lesson by the end of their first semester of teaching. While they were asked for their best number and operations lesson, two teachers named concepts outside of that mathematics strand. Teacher A named a geometry lesson and Teacher D named a measurement lesson. However, their responses were analyzed to determine which pedagogies were utilized during their “best” lesson. None of the memorable lessons were specifically from the district’s Math in Focus curriculum or from their mathematics methods coursework, but all of the lessons utilized either manipulatives or models as pedagogies.

Teachers B and G had students modeling solutions to multiplication problems during their best lesson. Teacher B gave students a word problem with missing information. The students were asked to draw arrays to model solutions to the problem on their desk. The teacher said she was worried the task would be too difficult and “go over their heads, but it really clicked.” She reported, “They were able to show me a lot of different strategies. Each person, probably the person next to them showed the same problem with the correct result, but with a different strategy. That was just kind of exciting."

Teacher G's multiplication lesson utilized a simple graphic organizer. A multiplication problem was written in the center of the paper. Students used skip counting, an array, a repeated addition sentence, and an equal groups picture to show multiple models of the given problem. This was an activity found on Teachers Pay Teachers which the teacher felt was successful with students. She used it multiple times because:

> They had four different ways to think about it. So I feel like I caught, hopefully, all the students. Maybe they didn't quite get the arrays, but they got the equal groups and they were able to think about it that way.

The other six teachers shared memories of a best lesson which utilized hands-on manipulatives of some kind. The following statements from teachers best describe which manipulatives were used, and how they were used:

- Teacher C - "They understood that nine is more than five because of this, and they understood the tower looks...I used cubes a lot so this tower is bigger than this tower so this has more."
- Teacher D - "I had one that was a fishbowl activity. They had a goldfish. They used a spinner. They spun it, and wrote down a number. Put that many goldfish in one bowl then they spun again. Put that many goldfish in another bowl and then counted their goldfish. They loved that game. It's so silly, but…"
- Teacher E - "I took paper clips. We were taking everything and measuring everything with it. It was very, very hands-on. It was fun teaching them how to use a ruler and how to look at inches. They got to go around and say the tissue box is this long...My favorite lesson was I'm going to lay on the floor and you guys are going to work together connecting cubes to see how long I am. Then they were measuring each other with cubes and blocks and how many pencils long is this? Anywhere they went it was, 'Oh, this is two jump ropes long. They just loved it."
- Teacher H - "The kids were able to build their houses based off of the number I gave them. They have to use their place value pieces (base 10 blocks) in order to make their

houses. Then I tell them to make changes to their houses and they have to take out two hundreds. They have to take out two hundreds and have to figure out what the new number is based off of what your house now looks like."

All the teachers' descriptions and justifications for choosing their best memories included comments that indicated the children were successful at understanding the mathematics concept being taught during the lesson. Best lesson descriptions from Teachers A, D, E, G, and H included comments about how much the children seemed to enjoy the lessons. Teacher A felt the students were "more engaged in math." Teacher C commented, "It was easy for me to find enrichment activities because they just got it so quickly."

Comments from Teachers B, C, and G seemed to indicate an increase in their confidence as a result of their best lessons. Teacher B stated, "Kind of like a moment like, 'Ok, this stuff really works.' My kids are really understanding this." Teacher C said, "They all did really good on that part which boosted my confidence up because they showed me that they are actually understanding something and I could see their improvement." Teacher G said, "There were a lot of lightbulb moments. It's always a good thing, and it makes you feel like you've done your job." Teacher H commented on her students' confidence levels increasing as a result of her best lesson. She stated, "At the beginning of the year, I just felt like their confidence levels were like, 'I can't do this. I don't know how to do this.' When we did that project it finally just clicked."

Analysis of the teachers' best lessons revealed additional patterns. For example, four teachers' best lessons were spontaneous lessons that either supplemented the district's curriculum or completely diverted from the textbook. Teacher A's shape best lesson built off of a textbook lesson. Teachers C, D, and E's lessons developed from class discussions or center time. Teacher D commented on how simple and silly her goldfish bowl center was, and how surprised she was at how much the children loved it.

It is also interesting to note that four of the teachers' best lessons involved content they were either most or least looking forward to teaching. Teachers F and G's best lesson matched the content that they were most looking forward to teaching. Teachers B and C's best lesson matched the content they were least looking forward to teaching.

The teachers were asked what they felt they needed in order to teach more lessons like the best lesson they had described. Their responses seemed to fit into two main groups. First, Teachers E and H commented on their need to feel more comfortable and confident in themselves as a teacher. Teacher H specifically wanted to understand the content more. The other six teachers' needs fell more in the category of preparation and planning.

Teacher B stated she needed more preparation time to plan out units, "It's really good to plan different questions and then also plan if there are ways or strategies that students are doing that aren't correct, and how do you respond." Teacher D wanted more collaboration time with colleagues and more training on the math curriculum. Teacher A wanted to plan more game based strategies. Teacher C needed to find more strategies to teach and assess concepts. Teacher F needed to find more "visuals and tangibles" to show students. Teacher G needed more "resources and examples" that she could use or adapt for use in her own classroom.

The teachers were asked to discuss their perceptions of the strengths and weaknesses of their mathematics methods course. The teachers' responses varied for both the strengths and weaknesses with only a few commonalities. Teachers A, B, and D felt their courses' strength was the introduction to different strategies and manipulatives which could be used with students. Teacher D felt a strength of her course was how the professor encouraged teachers to build a positive classroom culture. These teachers' statements highlight their courses' strengths:

- Teacher D - "I think cultivating an environment for kids where it's not so much about getting a correct answer. I think it kind of taught me a new perspective which was kind of nice because I thought of math as how I got math when I was a kid."
- Teacher B - "I just remember my peers in my methods course were pretty skeptical especially of the open ended prompts or tasks, high ceiling, low floor and inquiry-based. They thought students would kind of get confused. So I think that overall, just any skepticism I had of like, 'I don't know if this will work.' I think it's proven that it does."

Teachers F and H felt a strength to their courses were the lessons helping them understand student struggles. Teacher F felt she was taught how to differentiate in a whole group setting, and how to explain mathematics concepts to students. Teacher H felt the professor going over common misconceptions of students was a strength although she commented on not remembering a lot of them now.

When discussing weaknesses of their methods courses, Teacher G commented that she wished she had had “more practice teaching certain concepts like fractions and multiplication.” Teachers A and F commented on the fact that their courses were geared towards middle elementary level mathematics. There statements were:

- Teacher A - “I feel like it was a lot of the middle grades, but I feel like it could have done more with the higher and lower grades. I mean we did some with building number sense and recognizing patterns quickly. Things like that, but I feel like a lot of it was those middle grades.”
- Teacher F - “For weaknesses, kindergarten math is preliminary math and the course was focused on a higher level. I didn’t get a lot of information on teaching young kids. We had like multiplication and upper levels.”

Teacher C shared a concern that the university courses focused on instructional technology. For example, having preservice teachers find apps and websites that could be utilized when teaching mathematics. Teacher C stated:

> A lot of courses were mostly apps, digital, iPads, tablets, but I was like what else can I do other than finding apps because this generation of kids they’re always on iPads and technology. There’s a lot of schools that don’t have availability to teach technology.

At the end of each interview, the teachers were asked what advice they had for future mathematics methods course instructors. The most common responses were to provide more modeling, more opportunities for practice teaching, and more instruction on differentiation. Teachers A, B, D, and F felt the coursework would benefit from instructors or visiting teachers modeling specific strategies or lessons. Teachers A, B, F, and G felt the instructors should provide time in class for preservice teachers to practice introducing a concept, trying a strategy, or introducing a manipulative.

Teachers C, D, and H felt that methods courses should offer more instruction and examples of differentiation to meet the diverse needs of students in class. Teacher C felt that teachers should be prepared to have low level students and students that need to be challenged. Teacher H felt that methods courses could be stronger if there was more discussion about how lessons can be adapted "towards different levels."

Two additional suggestions for improvement were raised by the teachers. Teachers A and H felt that studying specific curriculum would improve methods courses. Teacher A suggested that the instructor model a specific lesson after the preservice teachers had studied the lesson in the teacher's manual. Then preservice teachers would understand how the lesson was laid out and implemented. She also suggested that preservice teachers could teach a curriculum lesson to a small group and get feedback from their peers.

Teachers E and G felt that more content should be discussed in the methods course. Teacher E suggested taking a week or two to go over content from specific grade levels because "If I walked into a sixth grade classroom right now I would have no idea how to teach that math at all." Teacher G suggested:

> Pick the biggest overarching concepts from each grade level...Just pick one thing from each grade level and do like a week or two focus on that and not so much on the lesson planning. I mean we knew how to write lesson plans by then.

CHAPTER 5. DISCUSSION

This study utilized interviews with eight beginning teachers during their first or second year of teaching elementary mathematics. This study sought to gather and analyze perceptions of their mathematics methods courses, and how the courses may or may not have affected their readiness to teach number and operation concepts in their first year of teaching. This study was guided by the following research questions:

1. What are beginning elementary teachers' perceptions of how well their mathematics methods course(s) prepared them for their first year of teaching number and operations?
2. What changes, if any, are there in beginning elementary teachers' perceptions of how well their mathematics methods course(s) prepared them for their first year of teaching number and operations?

Interview questions then narrowed the focus of the teachers' perceptions to three areas - attitudes and confidence, mathematical content knowledge, and mathematical pedagogical knowledge.

In the area of attitudes and confidence, teachers were asked to share their memories of experiences with growth mindset research and practices through their methods course, and what if any of their own classroom talk and practices aligned with growth mindset. Teachers were asked to rate their confidence to teach number and operations at the start and end of their first semester of teaching. They were also asked to explain the reasons for their ratings.

In the area of mathematical content knowledge, teachers were asked to share their content area background and attitude toward mathematics. They were then asked to share their perceptions for how well their mathematics methods course(s) prepared them in the area of content knowledge. Teachers were asked to share the content area lessons they were most and least looking forward to teaching, and to explain their thoughts on those choices. By the end of the semester, teachers were asked to share reflective perceptions for how they prepared for those lessons and how the lessons went.

In the final area of mathematical pedagogical knowledge, teachers were asked to share their memories of constructivist pedagogies introduced, modeled, or experienced in their mathematics methods courses. Teachers were asked to share descriptions of their own typical mathematics lessons at the start and end of their first semester of teaching, and to share their perceptions about their best mathematics lesson. Teachers were asked to explain their choices, and to share what they believed they would need to be able to teach more similar lessons. Teachers' responses were analyzed to see what pedagogies they reportedly implemented in their first year teaching practices as residual learning from the methods course(s). At the end of the interviews, teachers were asked to share their perceptions for how to improve mathematics methods courses by offering advice to methods course instructors.

5.1 Findings - Perceptions of Preparedness

While most findings from interview analyses fit into the focus areas of attitude and confidence, content knowledge, and pedagogical knowledge, three general findings are discussed here. These findings may be important to consider as they are likely to affect the perceptions and experiences of preservice teachers enrolled in future mathematics methods courses.

The first finding was that the teacher preparation afforded by mathematics methods courses varied greatly even over a relatively short time period at the same university. When I began this study, I anticipated that most, if not all, of my participants would be graduates from the same program. I feared this would lead to very similar, predictable, and frankly boring responses from the participants. However, while all participants did graduate from the same local university over a two year period, the eight participants shared differing experiences, memories, and perceptions of their methods coursework. The three participating teachers who graduated with master's degrees and the five undergraduate participants had different instructors and differing assignments, class structures, and experiences in their mathematics methods coursework. This seemed to support findings from Greenberg and Walsh's (2008) study of teacher preparation programs in the United States. Their work was entitled *No Common Denominator* because of the vast differences they found among programs. It appeared the title was applicable to this local university's preparation program as well. As Cochran-Smith and Power (2010) noted, teacher preparation faculty do have academic freedom of instruction which can make it challenging to create consistent experiences for preservice teachers.

A second general finding was that teachers didn't seem to know educational terminology. Some teachers needed a quick definition of growth mindset. None of the teachers knew or fully understood the term constructivist, and some teachers were unsure of which concepts fell under the number and operations strand. This is particularly interesting since all eight teachers referred to time spent studying the standards in their methods courses. The two teachers who mentioned learner styles made errors in reporting the three styles, leaving out auditory learners. Some teachers had difficulty naming specific manipulatives as well. Teacher H described geometric pattern blocks, but didn't know their name. Teacher A mispronounced "kee-sunaire" rods for use with fractions and couldn't remember the name tangrams. Teacher G referred to the educational terminology as "buzz words." She explained:

> Anytime somebody asks me what __ is, or what do you do for ___, I'm like, explain that. They explain it, and I'm like, oh, ok. Yeah I do this… I just don't know what to call it.

The third finding was that first year teachers' backgrounds and route to teaching varied greatly as well. Before I began this study, I had an erroneous, stereotypical view of first year teachers. I pictured a young adult of about 21 who had graduated from a four year program with a bachelor's degree. In fact, of the eight participating teachers in this study, only Teachers A, B, and H matched that description. Teachers C and G graduated with a bachelor's degree over a five year period with some transfer credits from other colleges. Teachers D, E, and F graduated with a Master's in Teaching after vastly different undergraduate work. Teacher E was the only one with undergraduate teacher preparation which she used to teach preschool for ten years. Teacher F, the only international student, was also quite a bit older than the other first year teachers and had previous classroom experience at Montessori schools.

Even at the same university, evidence indicated differences in the preparation preservice teachers received in the area of elementary mathematics. There was also apparent diversity among the preservice teachers coming to teacher preparation programs whether at the undergraduate or graduate level. This diversity may even increase in the coming years as states and school districts seek to fill teacher vacancies. These findings may greatly affect the perceptions and experiences of future preservice teachers enrolled in methods courses, and they may affect their perceptions of preparedness to teach number and operations concepts in their first year of teaching. For example, older preservice teachers with children of their own may perceive themselves more prepared than younger preservice teachers even after completing the same mathematics methods courses. Preservice teachers in the same program who experienced different methods instructors or instruction may also hold different perceptions of preparedness to teach mathematics.

For the remainder of this chapter, I discuss findings related to the two research questions in the areas of attitudes and confidence, mathematical content knowledge, and constructivist pedagogical knowledge. I also discuss how these findings may contribute to our understanding of first year teachers' perceptions of and experiences in mathematics methods courses and their first year of teaching elementary mathematics.

5.1.1 First Year Teachers' Attitude and Confidence

The first finding in this category is that math methods coursework was perceived to have positively affected multiple preservice teachers' attitudes towards mathematics and changed attitudes about how mathematics is taught. This seemed to match the findings of Saran and Gujarati's (2013) study. Teachers A, B, D, E, and F shared specific examples of how their mathematics methods course improved their attitudes towards mathematics. Teachers A and B seemed to have adopted the growth mindset in that they reportedly enjoy the challenge of teaching mathematics. Teacher E shared why she felt her methods course experience completely altered her attitudes towards math, changing from a person who hated and feared mathematics to a person who excitedly taught mathematics first thing every morning.

Five teachers commented that they were teaching mathematics differently than how they learned it. Their comments seemed to suggest that this change was introduced in their methods courses. They made comments that suggested the current trends of manipulative use and encouraging what Boaler (2016) termed mathematical mindsets, with an emphasis on conceptual understanding over procedural knowledge, improved their attitudes about learning and teaching mathematics. The data collected also seemed to suggest that the majority of teachers' goals are to teach for conceptual understanding in their own classes which may have resulted from methods course instruction. Kajander's (2010) study found that methods courses that stressed conceptual understanding yielded teachers with generally stronger beliefs in the importance of conceptual learning.

The second finding was that teachers seemed to perceive students' attitudes and emotions as important to the learning of mathematics. As a result, they seemed to show evidence of growth mindset by implementing growth mindset talk even if they were not using the specific terminology. Teacher F commented on helping children "have a positive experience in a math lesson and it will make them do better next time." Six teachers made comments about handling student mistakes and frustration and helping students see how mistakes and effort help them grow and learn. Only Teacher D gave specific examples of growth mindset grading practices she had implemented in her class. Teacher D noted, "I mean I wish I was a kid now in school by how much kinder and understanding teachers are. Just really focusing on what's good for the child and not stressing out the child." Teachers B, E, and G gave examples of mathematics tasks that allowed for multiple strategies and possibly multiple solutions which Sun (2015) felt conveyed growth mindset messages to students. These implementations were perceived to have been encouraged by their mathematics methods course.

Data collected indicated that all teachers' confidence ratings were shaped by knowledge of content, students, and pedagogy. If they perceived a weakness in one or more of these areas, the teachers assigned themselves lower ratings. All teachers' confidence ratings were at a three or higher, at some point, by the end of the first semester. Teacher E's confidence rating shifted depending on the content being taught. However, the other teachers confidence improved with knowledge of their students and with comfort in delivery of the district's textbook or supplemental materials. None of the teachers listed their mathematics methods course as a cause of their end of semester confidence ratings or changes in ratings.

5.1.2 First Year Teachers' Mathematical Content Knowledge

The first finding in the area of content knowledge was that the mathematics methods courses reviewed Common Core standards. The participating teachers seemed to view the study of these standards as content review, and multiple teachers referenced the benefits of this study. Both Teachers F and G commented on the benefits of studying vertical alignment. Teacher G even commented that part of her increased confidence was a result of studying what her students had previously learned while planning for the fractions lessons she was least looking forward to teaching.

The second finding was that data seemed to indicate that neither mathematics methods courses were perceived as adequate review of mathematics content. The negative perceptions shared by the five teachers who took the mathematics department content course seemed to indicate that the course did not improve their content knowledge. The teachers' comments also seemed to indicate that the course was not perceived to be useful in preparing them to teach number and operations concepts. Teachers A, B, and C had multiple mathematics courses in high school and at the university. They did not attribute their content knowledge to this content course. Teachers G and H reported mathematics as a weakness during their first year of teaching even after taking the two required mathematics methods courses.

Perceptions of the mathematics methods course itself were slightly more positive in the area of content review. While the data indicated that most teachers felt that content was not adequately reviewed, multiple teachers did perceive a better understanding of the computation algorithms discussed in methods course. For example, Teacher G remembered discussing the traditional subtraction algorithm.

5.1.3 First Year Teachers and Constructivist Pedagogies

The first finding in the area of pedagogical knowledge was that hands-on manipulative use was being encouraged, but implementation in methods courses varied. Teacher B described her class's seating arrangement as tables with the day's manipulatives stacked in the middle. All other teachers remembered only a few instances of manipulative use in the actual class. Multiple teachers stated that they used manipulatives and saw manipulative use modeled more in the field practicum placement than in the actual methods course.

Hands-on manipulative use increased by the end of the first semester of teaching. Six teachers reported an increase in the different types of manipulatives used. Teachers B and H had reportedly only used base 10 blocks by the second interview, but planned to use additional manipulatives when they started fraction instruction after the winter break. The most frequently reported manipulatives used by the second interview were base 10, counters, and unifix cubes. All five lower elementary teachers used either counters or unifix cubes by the second interview. Geometric pattern blocks were used by three of the five lower elementary teachers despite the fact that all five teachers taught the same content based on the district's pacing guide and adopted curriculum. It is unknown whether the other two teachers used the manipulatives and didn't report it, or if they taught the geometry lessons without manipulatives.

The third finding in the area of pedagogy was a result of data collected about teachers' typical mathematics lessons. While all teachers reported some changes to the structure or pedagogies used in typical lessons by the end of the first semester, the teachers' contexts were perceived to be a barrier to mathematics methods course residue. None of the teachers made negative comments about their coworkers or school climate. However, multiple teachers commented on their perceived need to use a specific curriculum and move through it at a predetermined pace. Participating teachers shared examples of how they had supplemented curriculum lessons through the semester much like the participants in Haggarty and Postlethwaite's (2012) study of beginning teachers. Multiple teachers also referenced scheduling issues as a perceived hindrance to teaching in small groups or with centers as they had been encouraged to teach in their methods course. This data correlated with Diez's (2010) reasoning of why new teachers may not be implementing aspects of their teacher preparation program.

Finally, first year teachers wished for more modeling and practice of pedagogies in methods courses. Multiple teachers reportedly wished for more modeling of pedagogy either by the methods course instructor or by elementary educators. Data collected also seemed to indicate that first year teachers felt finding ways for preservice teachers to practice pedagogy in the methods course would not only improve the course itself, but would also improve their teaching in the first year.

Overall, beginning teachers' had more positive perceptions of how well their mathematics methods course prepared them in the area for teaching number and operations concepts in the areas of attitudes and pedagogies. The teachers' perceptions seemed to indicate that they just wanted more pedagogy modeling and practice. They had less positive perceptions of the quality and quantity of mathematical content reviewed in their methods courses.

5.2 Recommendations for Mathematics Methods Courses

From the comments collected in this study two recommendations are suggested for improving the teacher preparation afforded by mathematics methods courses. The first recommendation concerns the actual structure of the coursework over the two semesters required at many preparation programs. The other recommendation involved the assignments preservice teachers complete during these methods courses.

The first recommendation that seemed to be supported by this study's data involved the restructuring of the two mathematics methods courses required for undergraduate elementary education majors. Instead of two separate courses focused on only content or only pedagogy, two new courses could combine the content and pedagogy each semester. This would offer more opportunities for preservice teachers to review or relearn elementary mathematics content as they experienced and practiced constructive pedagogies appropriate to that content. This advice would likely be supported by multiple researchers who have found positive effects of integrating content and pedagogy in mathematics methods courses (Ball, 2000; Beal, 2001; Burton et al., 2008; Fast & Hankes, 2010; Ford & Strawhecker, 2011).

In order to ensure the mathematics content is taught to university standards, the courses could be co-taught by mathematics and mathematics education department instructors. The combined comments from the five participating undergraduate level teachers seemed to show that the content level course did not fulfill its intended purposes of content instruction. Multiple teachers shared negative perceptions of the mathematics department instructors because they couldn't understand them due to language barriers with international, visiting professors or the professor's inability to present the mathematics at their level. Many teachers also commented on the fact that the professors modeled teaching techniques that were contrary to what they were being taught in the college of education.

It would appear that this semester of study could be better utilized by the college of education to improve teacher preparation in the area of mathematics. Teachers A, B, and C, who had a mathematics background, were dissatisfied with the content review course. Teacher B says she read a book during the course because she didn't see the point of it. Teachers G and H, who felt mathematics was their weaker area, didn't feel their content knowledge improved through taking the course. Teacher G's story about the class's grading curve yielding her 69% average a B for the course, seemed to prove that others didn't experience mathematical success either.

In contrast, all of the participating teachers had positive comments about their pedagogy instruction, but many commented on the brevity of the instruction and the mostly non-existent content review. This is not necessarily a fault of the methods instructor or the preparation program. They assumed the content review was adequately taken care of by the content coursework. However, as Fast and Hankes (2010) discovered in their study, if preservice teachers didn't understand the mathematics concept they often missed or questioned the effectiveness of the pedagogies introduced in class. Teacher B referred to the skepticism of her methods course peers who thought the inquiry and open ended mathematics tasks would be too confusing for students.

Many of this study's participants said they couldn't see how there could be enough time to add improvements to a one semester course. If the college of education restructured the two semesters to break the elementary content into two sections, then the content itself could be reviewed while effective pedagogy of that content was modeled and experienced. There are two possibilities for how to restructure the content - sections determined by elementary grade levels or sections determined by mathematics strands.

In the first scenario, semester one could focus on elementary mathematics content as taught in the early elementary grades of kindergarten through second grade. Semester two could focus on the content taught in grades three to six. This may be mostly beneficial to those preservice teachers who already desire to work in the primary grades. The kindergarten and first grade teacher participants perceived that their courses didn't provide enough preparation in early number sense and beginning mathematics concepts. However, this type of restructuring may cause more problems for those preparing to teach upper elementary grades. Computation concepts and algorithms in the upper grades are numerous and challenging. This content may still require more time than a single semester methods course.

Most likely, the better option for restructuring methods courses may be to use the specific strands of mathematics as a way to separate the content and its pedagogy into two semester long courses. The first semester methods course could focus on number sense and the concepts and algorithms of whole number and decimal addition, subtraction, multiplication, and division. Algebraic reasoning and patterns could easily be integrated in the instruction of these four operations. If these newly structured courses were team taught then mathematics department professors could review content and be on hand for clarification of misconceptions, definitions, concepts, and procedures. Mathematics educators could model and provide practice of instruction using constructivist pedagogies. This structure may even have the added benefit of improving the mathematics professor's own instruction in their future courses.

The second semester methods course could focus on fraction, data and measurement, and geometry concepts. Fraction instruction could include fractional number sense, the four operations, and decimal equivalence. Again, algebraic reasoning and patterns could be integrated in these content areas. Knowing the specific mathematics strands covered in each course could determine which mathematics department professors were asked to co-teach the lessons. Ultimately, the mathematics department professor's disposition may be the more important criteria by which to choose a co-teacher for a methods course. The professor must see the value of preparing preservice teachers to be effective mathematics teachers. Professors with expertise in rational numbers or geometry could come in for lessons covering those specific topics. This may increase collaboration between the mathematics and mathematics education departments.

Restructuring by mathematics strand could provide preservice teachers with a better understanding of the developmental growth of elementary students and vertical alignment of K-6 standards. Seeing how students "grow" in each area of mathematics can also provide the information preservice teachers need to differentiate instruction for their future students because they will know how to remediate or enrich instruction of the mathematics concepts.

The second recommendation that seems to be supported by this study's data involves changing the assignments required in the methods courses to allow for more in class pedagogical practice and reflection. Many of the teachers in this study commented on the need for more practice even though they completed a practicum experience in conjunction with the course. Teacher G commented that since they had to teach whatever the practicum teacher needed, there wasn't a lot of "connectivity" to the methods course. Other teachers also commented on the fact that they had to follow the adopted curriculum at their field placement. Relying on cooperating teachers to provide the majority of practical experience may be risky if those teachers and/or their teaching context does not encourage the same philosophies and pedagogies as the preparation program methods courses.

Multiple teachers in this study suggested that they'd like to see more modeled lessons and have more practice before teaching independently. As shared in the literature review, the preservice teachers in Basturk & Tastepe's (2015) and Unlu's (2018) studies showed positive effects of micro-teaching experiences in methods courses. In this study, Teachers D, E, and F all commented on how much they learned by watching and giving feedback on their own and others' videotaped lessons. Videotaping lessons could provide both the practice and modeling this study's participants suggested when giving advice to future methods course instructors.

Multiple researchers advocate the need for teacher preparation programs to be practice-based (Ball & Forzani, 2009; Chan & Franke, 2010; Cochran-Smith & Power, 2010; Cochran-Smith et al., 2015; Conrad & Tracy, 1992; Darling-Hammond, 2010; Strawhecker, 2005; Youngs & Qian, 2013). Chan's (2010) dissertation study also showed that when preservice mathematics teachers in practice-based mathematics methods courses had what she termed a "trajectory of participation" (p. 18) they improved their ability to implement strategies learned in the course. The trajectory she described began with the preservice teachers using the strategies in the methods course, then practicing the strategy in a university lab school, and then in their student teaching experiences. She suggested that this could lead to an increase in the teachers implementation of the strategy in their own classrooms.

5.3 Directions for Future Research

Researchers have suggested that program graduates' feedback is essential for improving program and individual course curriculum (Chigeza et al., 2017; Ensor, 2001; Harris, 1991; Linek et al., 2012; Valli et al., 2001). When changes are implemented to programs and courses, it would likely be important to continue soliciting corresponding input from new graduates.

Future research could be done at universities willing to restructure their methods courses as recommended in the previous section. Research should continue to determine the extent to which first year teachers implement the philosophies and strategies encouraged by their preparation programs. I argue that if the beginning teachers can't remember the lessons, they are unlikely to implement them. Therefore universities would benefit from conducting follow up surveys with graduates who have finished their first year of teaching to determine the level of residue. There are important questions to consider - Are teachers doing what they've been taught? How can first year teachers be encouraged to revisit their methods learning whether it be reviewing texts, articles, or lessons in their portfolios? Were the portfolios created just as an assessment piece for the student teacher, or are they a portfolio of lessons that can be implemented in the future?

While manipulative use was reportedly encouraged in all methods courses, the introduction, modeling, and frequency of manipulative use varied between courses. This raised questions that could be the basis for future studies. Are methods courses relying on cooperating teachers and practicum schools to supply manipulatives and opportunities to practice teaching with manipulatives? Are beginning teachers more likely to use manipulatives if they've experienced and practiced teaching with manipulatives in methods courses versus in practicum settings? Are beginning teachers more likely to use manipulatives if they graduate from a well-supplied teacher preparation program? What happens to preservice teachers who have cooperating teachers that don't use manipulatives in their own classrooms?

Barriers to implementation of preparation program learning should be identified, and further research should address ways to minimize or eliminate those barriers. In this study, multiple participants noted that they felt the need to follow the district's adopted mathematics curriculum and the district's pacing guide. At first, many did not deviate or supplement that textbook's lessons even when they felt it contradicted the methods course learning. Teachers A and B even commented on the fact that they wished the methods course better prepared them to use the textbook curriculum. Teacher F put aside years of training and previous experience in hands-on instruction to follow a textbook and pacing guide. By the end of the semester, the teachers were starting to make their own instructional decisions within the textbook lessons. This is a necessary and desirable aspect of instruction as teachers seek to meet the needs of their students. Further research may want to study the pressure teachers feel to comply with practices that go against their personal beliefs and preparation, and the potentially negative impact this may or may not have on teachers. Teachers D and H in particular made comments that seemed to imply they felt guilty about not following the textbook lessons even though they reported that they and their students seemed to enjoy and benefit from the lessons.

5.4 Limitations

This study relied on the memories of beginning teachers who responded to personal or emailed invitations to participate in a doctoral study. The reported perceptions were not verified or confirmed. Therefore, certain limitations to this study's findings are recognized here.

Emails were sent out to all first and second year teachers in the researcher's local school district. Personal conversations between the researcher and first and second year teachers at the researcher's own elementary school were used to solicit volunteers as well. Although three teachers were vocal about their weaknesses in mathematics, it was likely that only teachers who were comfortable with mathematics would be inclined to answer a stranger's email for participation in a study involving mathematics.

The attitudes and confidence reported by these participating teachers were likely influenced by a complex combination of multiple experiences. This could include childhood and adolescent experiences in their own mathematics education, their preparation coursework, cooperating teachers, and practicum experiences. Perceptions of preparedness may have been negatively influenced from the stress beginning teachers naturally feel their first year of teaching through no fault of their methods courses. In addition, the participating teachers' current colleagues, coaches, and mentors met in the first year of teaching are likely to have had an impact. The context of their teaching assignment could influence their perceptions about their preparation program. The teachers' perceptions are based on their understanding of people and events and personal beliefs which this researcher must take at face value. Although it was not and cannot be fully verified, it was assumed and hoped that the participants were honest and open about their perceptions and memories of their methods coursework.

Participants were not given the interview questions ahead of time even though they were asked during the interviews to recall information from two or three years into the past. This was done because of the researcher's interest in determining which aspects of the methods coursework were most memorable by and even during the first year of teaching mathematics.

There are obvious disadvantages to the passage of time and lack of advanced interview preparation. It is possible that the teachers have forgotten important methods coursework for a variety of reasons. The time between the methods courses, student teaching, and the first year of teaching combined with the stress, hectic schedule, and steep learning curve of these experiences may have had a negative impact on the teachers' ability to remember what they were taught. In addition, as Teacher G noted, when preservice teachers are in the classes they don't really know what they'll eventually teach. They may not accurately predict what is important and worth committing to permanent memory. Therefore, it cannot be assumed that these teachers' memories accurately describe their courses or their instructors.

The perceptions and memories of the participating teachers were not verified. Course syllabi, assignment portfolios, or contact with the course instructors could have been used to verify whether the methods courses did or did not cover certain topics. However, the accuracy of the beginning teachers' memories was not the focus of this study. The focus was teachers' perceptions of their methods courses and how prepared they perceived themselves to be in terms of teaching number and operations concepts. This study sought to determine the residue from the participants' methods courses. Therefore, readers of this study should not assume that certain topics were or were not taught in this university's methods courses. Readers are limited to participants' perceptions of the courses only. The perceptions cannot be generalized to methods courses taught by other instructors at the same university or at other universities.

5.5 Concluding Remarks

Research suggested that preservice teachers may come to their elementary preparation programs with weaknesses and anxiety in mathematics. As teachers play a crucial role in student achievement (Hattie, 2009), it is important that research include the study of teacher preparation. This study provided a way to understand the experiences of first year elementary mathematics teachers by gathering their perceptions of readiness to teach number and operations concepts which makes up the majority of elementary mathematics curriculum. Eight teachers were asked to share perceptions of their mathematics methods course and how it influenced their attitudes and confidence, mathematical content knowledge, and constructivist pedagogical knowledge.

Under each of these three sections, the focus was narrowed down to specific aspects of teacher preparation that I wanted to understand better. My focus was on the teachers' attitudes and anxiety in mathematics during their first year. I wanted to determine the extent to which they had been exposed to growth mindset in their methods courses because I theorized the exposure might have positive effects. I also hoped to see evidence of mindset in classroom practices. Next I wanted to focus on how their methods courses utilized constructivist methods to affect their preparation in the areas of content and pedagogical knowledge.

Data collected indicated that all teachers perceived themselves to be positively affected by their methods coursework in the area of attitude, and all teachers were encouraged to use constructivist pedagogies. The majority of teachers felt their content knowledge was not greatly affected by either methods courses. This study seemed to support the findings of Greenberg and Walsh (2008) that mathematics content is missing or weak in methods courses. The authors also found that mathematics department content courses had less demanding content or expectations, and were taught by newer or visiting faculty which this study's findings seemed to support.

The data collected during interviews seemed to support the findings of Cochran-Smith and colleagues (2015) that first year teachers struggle as they transition to independent teaching. These teachers shared first year challenges such as relearning mathematics concepts and vocabulary by themselves, learning a new curriculum and its components, and the perceived stress of following pacing guides and schedules. These struggles took place while they worked with students of a wide range of abilities or students with poor attitudes towards mathematics.

Kastberg et al. (2013), Darling-Hammond (2010), and Valli et al. (2001) found that many first year teachers were not practicing what they'd been taught in their preparation programs, showing low levels of what Kastberg and colleagues (2013) considered residue. This study also found examples of instances were teachers' choices had been influenced more by their school or district's context than their preparation program. This seemed to show a disconnect between the preparation program and the education system, however the question to be answered is, Who's most at fault for this disconnect? Is the university program not preparing the teachers accurately, or is the educational system deviating from research and teacher preparation work? This is a weighty question that cannot be answered in the scope of this study, but it is through further study that we may be able to find solutions for the system itself and the teachers.

This study helped identify opportunities for improving mathematics methods courses specifically by asking beginning teachers for their perceptions of their preparation and for advice on improving the courses. This feedback could be used to inform best practices for methods courses by determining which aspects of the course were memorable and readily applied to teaching number and operations concepts even in the first, stressful year of teaching. If feedback were solicited from beginning teachers, methods course instructors could, as Kastberg and colleagues (2013) suggested, study and share best practices to reduce variability in the courses.

This study contributed to my understanding of how mathematics methods courses may or may not impact first year teachers' confidence, content knowledge, and pedagogy. Creating methods class activities and assignments that will leave a lasting impression seems to be the challenge for methods course instructors. These activities and assignments must not only help preservice teachers appreciate the power of constructivist methodology, but must be transferable to independent classroom practices during the teachers' first year. The time between the methods course and the first year of teaching adds to that challenge. However, studying the first year experiences of these eight teachers improved my ability to draw connections between mathematics methods course preparation and teachers' classroom practices. Examining the residue, or memorable aspects, of their methods courses can shape the curriculum of future methods courses I hope to design and teach.

In this study, I argued for the application of mindset and constructivist methods because I think preservice teachers need to be shown the effects of those methods in order improve the chances of transfer to classroom practices. I believe that you cannot just tell preservice teachers, "This is how you should teach mathematics. Trust me. It works, just do it." That style of teacher preparation resembles how mathematics used to be taught, "Here's the formula. Trust me. It works, just use it." Mathematics educators now know that doesn't effectively teach mathematics so just telling preservice teachers doesn't effectively prepare them to teach mathematics either.

I agree with Amirshokoohi and Wisniewski (2018) who stated, "The aim of providing teacher candidates with such personal experiences is for them to recognize how such learning experiences may enhance children's interest in mathematics and diminish math anxiety" (p. 448). My goal is to create methods courses that allow preservice teachers to build their content and constructivist pedagogical knowledge, and feel prepared to implement what they've learned. I believe such methods courses can influence mathematics classroom practices even during the stressful, first year of teaching.

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APPENDIX

Sample Sorting Charts from Typological Analysis of Study Data

Beginning of Year Ratings of Confidence and How They Determined Their Rating

Rating based on concerns for content knowledge and knowledge of students	HI1-5 H – probably like a 1 or a 2 K – ok. Tell me how you got that rating. H – I feel like when I interned I felt like my cooperating teacher she was really from the Math in Focus curriculum and I didn't really understand how to teach from that curriculum. It was just like I had to really reteach myself the skill, but not only the skill but how they do it in the curriculum. She was teaching right from the book and I was like oh, man. I'm going to have to follow this. I'm going to have to figure out exactly what they mean. Then I think I only taught multiplication in my internship which I really love multiplication. It's like my favorite to teach so that was the only thing I was really confident in. When I graduated, that's when she started teaching fractions and everything else. We used the first semester on those first chapters up until division and that was all I saw. So then thinking about oh, man I'm going to have to do all this by myself. I haven't done fractions in how many years… I haven't seen any of the curriculum so I was just I'm seriously going to have to reteach myself all of this. I just haven't seen it. I haven't seen how the kids will do, what they did in third grade. I didn't know how they would progress and where they would be once they got to me. GI1-4 G – I feel like I was still pretty confident as a first year teacher just because I had done my internship in the same grade level. I had already done the same things with my teacher last year that I was going to be doing by myself this time. So it was stressful. I mean I wasn't as perfect as if I'd watched my cooperating teacher, oh yeah that's what I need to be doing I don't always do it the exact same way so I would say maybe a 4 or a high 3. Because it's the beginning of the year and 3rd grade has a lot easier things to start with too. If I'd been starting my internship, not having that yet, it would have been like a 1. It was not that comfortable. K – being in the same grade level as your internship as your first year of teaching… G – Yeah, that definitely helped. If I'd been at a different grade level I'd definitely been a 1 or a 2 because I would have felt lost.
Rating based on knowledge of students at a particular grade level	CI1-4 I was probably at a 1. I wasn't very confident because it was my first year, and I was terrified. It's kindergarten and I never been in a kindergarten classroom because I was always in a first grade to a third grade classroom. I didn't know what kindergarten was like, and I was actually hired the first week of August. So I didn't really get into the curriculum until basically a week and a half before the first day of school. I didn't get that chance to actually look into the curriculum and dig deep into it. I was stressed out and scared all at the same time. K - Even though you had a strong math background? You'd taken calculus, and trig, and all these things. So what part of it really stressed you out then? C - I think it was because I didn't know how kindergarten was like and what their number sense was like. Because they don't really do addition in kindergarten so I was like what do they know? EI1-4 E – I would say I was at probably at a 2 or a 3 because I was really scared. I had that part of me that knew I could teach it, but also I didn't know what kind of kids I was going to have. I didn't know what kind of learning disabilities they were going to have. I didn't know if I was going to reach them, or teach them in a way that they would understand. So I was really, really nervous." DI1-5 "D - I would say about a 3. I feel like I have a lot of tools in my toolbelt. What I don't feel like I was prepared for was being in a Title I school and having kids that are low. That are coming to me that first unit, that first chapter is such a struggle so I'm feeling like I need to go backwards a little bit. Also, trying to keep up with the pacing guide is very hard because I feel

	like my kids...They want me to spend two days on a number line and I need a week on number line. So feeling like I'm covering all the content and at the same time adapting it so it fits the needs of my students - that's my biggest struggle. Feeling like I have so many students that need that one on one, individual teaching and not having the resources to do that. "
Rating based on knowledge of mathematics pedagogy	BI1-2 "On a scale from 1 to 5, I would say a 3. K - A 3? Ok, and what made you give yourself that rating? B - I felt like I had a decent amount of tools and then I practiced a little student teaching in first grade and they're way into number sense so that kind of helped too but I think that my, like, lack of confidence would have been in how to implement it. Kind of like I know what my goal is that math discourse and sequencing and everything, but how to get there I feel like I don't have like the skills or experience to really do that." BI1-2 " just would love for that student discovery to happen instead of student memorization or me telling. I think trying to facilitate those discussion and have it be really purposeful for the majority, well obviously all students, but that would be like that's the goal and I think to do that well I don't feel like I'm not super confident in that, but in the content I feel fine." BI1-2 "I feel like I have tools to create discussion in the fractions." BI1-2 "I feel least prepared where we're going to transition into multiplication next and I think I feel least prepared skills-wise, like teaching strategy-wise, on that one. To really start it off inquiry-based so they are discovering why and what multiplication means. To really facilitate good discussions with introducing multiplication and division, I don't feel like I have a ton of skills on that." AI1-4 "I would probably say probably 3.5 or 4. I don't know. I taught some of it, not necessarily these. I've always been more confident with math just throughout school and stuff, but once you get to the regrouping and borrowing that's when I think it gets challenging because I know how to do it but explaining it is like a whole nother ball game. So I would say that especially with these first two chapters with number bonds and numbers to 10 it's been pretty simple and I've felt pretty confident with it but I know some of the chapters where it gets a little more complex I might need to think of other resources and really sit down and look at it more and figure out how I can come up… give the information to them in a way that they're going to need to truly understand it." FI1-7 " I would say 4. I was just not sure about the pace that I will go, but knowing content and what I need to use to deliver the content, I was feeling pretty confident." FI1-8 How am I going to give them enough time to catch that up? If they don't catch up, yes they will have some time to catch up in first grade, but then first grade content will be slowing down there. There's not enough time for me to have one on one connection with them to see how they're progressing. That's still my weak point, I have to say. I am used to teaching one on one because Montessori lessons, with math especially, they are mostly one on one or small group of 2 or 3. So I'm able to get the feedback pretty easily and know where they're at pretty easily. But with a whole group, I have to get used to it. I will get used to it and have my eyes on those kids who are struggling down there. I think that will be my weak point."

Evidence of Growth Mindset at Start of the Year

Memories of Growth Mindset from Courses	**No use of term "Growth Mindset," but felt some aspects of GM were encouraged in math methods course**	BI1-1 "K - Was growth mindset ever brought up or talked about in that class? B - Um a little bit, it was brought up in the college of ed but not so much in the math class" GI1-3 "G – I don't know that we had talked about growth mindset in that class. We talked about different learners, even conceptually younger kids even if they're just younger for their class sometimes conceptually they're just not at the same step they just can't cross that threshold of abstract to concrete. The difference in what they're trying to figure out. It wasn't specifically growth mindset it was trying to cater to those learners you're going to have for differentiating." HI1-4 "K – Did he broach, or how did he broach anything about growth mindset? That's a fairly newish topic. H – He didn't really mention anything about that. K – Do you think in any way that he might have encouraged that growth mindset even if he didn't use the phrase? H – I guess, you know, if you don't get it just keep trying different methods. Keep trying different strategies until you get to the end result that you want. I think that maybe that was his reasoning to get the kids to keep trying to find ways that work for them." CI1-8 "K - Back to your math methods course, your block c course, do you think it encouraged a growth mindset about mathematics? C - Yes because even though they would teach different strategies, we had different points of view and how kids were thinking in their mindsets and their thinking process it kind of let us be open about our different points of views on things. It let us open up about different strategies and how we can improve ourselves and how to teach something and how to get better at assessments." DI1-8 "I even remember, when I was working with my host teacher in student teaching, she said it's so much more important to me that I see their work, that I see what they did, and their strategies, then that they got this answer. Teaching that to kids because that is more of a life skill and that's the mathematical mind that you want to build. We're not after one answer, we're after how can we solve this problem." GI1-11 G – I think so. I know we talked about a lot going back and reviewing, knowing that it's ok. You're not always going to get stuff right. That's just kind of in general I remember talking about that in all kinds of courses. I feel like our teacher just made it ok, she said you make mistakes. Show them that it's ok to make a mistake. Purposely mess up and make them correct you. Show them it's ok for that to happen and or to correct each other. It kind of more specifically focused on that. I know that growth mindset is more than just "it's ok to be wrong". That's what I remember. What I relate that to growth mindset.

	Use of term Growth Mindset in courses as related to classroom talk and attitudes	AI1-3 "Definitely growth mindset. Our professor, we did, I don't know if you'd call it a unit, but we did several activities where it was growth mindset. She said how you might not like my class because I'm going to make you work hard and really break this down and think about how to teach your kids. We just talked about why we don't want to give up." DI1-4 "The course was teaching us that the most successful kids in math are the kids that have been taught that they're allowed to make mistakes, and that mistakes is part of that growth." "So that math methods course is designing lessons that are challenging enough to engage your kids and to teach your kids that's it's ok if something's hard." FI1-4 "K - Absolutely. Now did that get introduced to you during that math methods course, or how did you know about that? F - We talked about growth mindset at the very first class. So that's how I knew, but before that my daughter, when she was 4th grade, her teacher had a big poster board called growth mindset. K - When the professor of your math methods course introduced growth mindset to you, what were some of the things that she said that stuck out to you? F - Math is really hard for kids because it's really, really abstract, but having this growth mindset in introducing concepts or work expectations and wording things differently like instead of saying you're wrong so you're getting a point off, say you've done 2 correct. Let's do better next time. It will help the children have a positive experience in a math lesson and it will make them do better next time."
	Use of term Growth Mindset in courses as related to specific classroom practices or procedures	EI1-3 "K – Did they expose you at all to the growth mindset research or talk about mindsets in math? E - What would be an example of that? K – Like persevering. I'm not good at this yet, but E – Oh, yes. One thing that was taught and I still do this today is taking out the words "it's so easy" because that can be really detrimental to children who that's not easy for. They taught us with grading, if you give them a worksheet with 5 problems and they get 2 of them wrong, right there they're getting a bad grade if they get 3 wrong. The way that they taught us to grade is to not necessarily just look at 4 + 1 is 6 and saying the whole thing is wrong. They missed that. It's looking to see the different ways, how do they get to the problem. Look at the different ways of solving it. Kind of grading to boost their self-esteem, I guess.E – Well then, for me, what really stuck with me was the different way of grading. Rewire your brain. I understand if a kid gets it wrong, they get it wrong. I understand that. If I look at their math problems and I see that they tried really hard for the whole time they were taking the test, but they only got half of the test done but I saw they were working hard and they were persevering and they were doing their best, I'm just going to not grade them for the stuff they didn't do and only grade them for the stuff they did do. So if it's out of 20 and they only do 10, I'm not going to say oh well you got 10 out of 20. I would look at those 10 problems and I would grade those 10 problems. They taught us a different way of grading to reward the children for the hard work that they do, and to not make them feel bad. Ok, you worked really hard and I can see that, but you just didn't quite get there."

<table>
<tr>
<td colspan="2">Evidence of Teacher's Own Growth Mindset towards challenges</td>
<td>AI1-5 "That's probably my favorite kind of math is when you get into adding and subtracting a little bit bigger numbers, but it's also probably my area where I'll feel most challenged with…
K - ok so you enjoy the challenge
A - yeah
K - so I guess you took on that growth mindset after all, huh
A - yeah (laughs)"</td>
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<tr>
<td rowspan="2">Handling Mistakes & Frustration</td>
<td>Using phrases specifically associated with growth mindset</td>
<td>AI1-7&8 "K - Do they ever say those statements back to you, like unsolicited?
A - Yeah sometimes I hear it. Because we talked about testing. "Testing was hard wasn't it?" "Yeah my brain worked really hard today." so yeah, they're picking up on some of that. We've done one lesson on like how we can't say "I can't do this" it's "I can't do it yet." and how some things are challenging for some and not for others, but we all have different skills and we're going to learn a lot this year so…
K - gotcha, so spreading that mindset
A - Trying to. It's hard. Some tend to give up easily when they're frustrated which I get but we're working on, you know, maybe I can explain it some way else so it doesn't seem as overwhelming or as challenging in your eyes."</td>
</tr>
<tr>
<td>Using aspects and phrases of growth mindset although not specifically associated with growth mindset research</td>
<td>AI1-7 " So what are some of the things that you do or say if somebody makes a mistake in here?
A - Your brains growing. We're learning."

BI1-4 "K - How do you handle the darlings that get frustrated in here - kind of like how you had the frustration in your methods class? What are some of your typical phrases or go-tos?
B - I think I mainly try to empathize. Like if they see, my biggest frustration so far, this is from a kid's perspective, has come from the subtraction with regrouping. They've been told to start with ones and the ones has zero minus nine… so it's like this is impossible so they get kind of frustrated. Then kind of empathizing like, "You're right. If we have zero apples how are you supposed to take away nine?" "

BI1-5 " I think with the frustration, trying to validate it first. Don't say, "Stop being frustrated, this is how you do it." It's kind of like, "You're right. This is weird. Let's see if we can figure out how to do it."

DI1-6 "K - When somebody gets frustrated at one of the centers, what typically happens - if that even happens?
D - It's something that I have to stay firm on because kids at that age, developmentally, their instinct is just to go to an adult. What do I do? Getting them comfortable with being confused and giving them strategies. If you're confused do this. If you're still confused do this. Giving them some specific concrete things to do when their first method doesn't work. Even teaching them, you got it the first time, here's a way you can check and make sure you're right. Things like that. Because it's such a learning experience when they're asking other students. It gives that other student, that's the greatest level of mastery is if you can teach it to somebody else."

DI1-7 "K - You're trying to teach math which can be a stressful subject for people.
D - and for kids. Kids in first grade have already said, labeled themselves I'm bad at math. Like, who told you that? How do you know that? Why do you think you're bad at math?"</td>
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		FI1-10 Do you talk much with your students about growth mindset? F - Not really. Just in between lessons, or in between the work, if they're frustrated I'll just say it's ok. You can do your best and show me you can do your best. That's all I want to see. K - Do you talk about the fact that math is sometimes frustrating? F - Yeah K - Do any of the kids say things like "oh, I'm not good at math." F - Not really. Not at this point EI1-12 K – So what are some of the things that you say to them when they're frustrated and crying? E – For instance, I teach an intervention group. We're just working on simple number lines and some of the kids… I would say what comes before 14 and they can't tell me. Ok, what number comes after 17? You can see they're getting frustrated so I just take a minute and think about how I'm explaining this and how can I do it differently. It's ok. Don't get frustrated. A lot of the things that I say, even if it's not true for them, is like "I'm not mad. You're not in trouble." I remember thinking when I was little I don't know this so I'm in trouble in the teacher's mad at me so I'd cry. So even if it might not be true for one of my kids in my class, I still say it's ok. I'm not mad. You will get this. Oh, I'm stupid. No, you're not. You're still learning. I always say this is why you're in school. You're in school to learn things. That's why I'm here. I'm here to show you how to do it, and teach you how to do it. I just keep re-explaining it as many different ways as I can possibly come up with. I think I spent 30 minutes on Thursday just teaching what number comes before 7? 30 minutes to a group of kids that just could not. Number lines weren't doing it, and they were getting really frustrated. I just kept saying it's ok. It's not a big deal. Just reassuring them because I didn't have that kind of reassurance. I just consistently let them know they are fine. They are ok. We will learn this. You're here to learn. No you're not dumb, you're smart. This is hard stuff. You're a first grader. I don't expect you to know all of this. GI1-11 K – Even if last year you didn't talk to your kids and use those buzz words, do you feel like you encouraged that attitude towards math in your class? G – I did do, I would purposefully mess up, whether it was addition or multiplication or whatever. I would do ok watch me do this problem. I would do it and have them check me either all together or in a small group maybe. If I could tell that a kid was kind of struggling, I would go up and try, and I'd let them correct me. It kind of gave them a boost like Ms. G got it wrong so it's ok. I can go and fix mine now. They could get stuck in the mud mentality with it. I remember writing on the board, trying to specifically mess up. I would pretend to move on and they'd be like no, no, no, wait and then tell me what I'd messed up on. Then I'd ask what do you notice? What did I mess up? Is there a reason why I got it wrong?
Encouraging Effort & Working Hard		BI1-3 " Ideally, I don't know how well I've done this so far, but ideally it would be, I try to celebrate guessing, so just the goal is effort not the end result. Celebrating different strategies a lot more so than right answers." CI1-6 I don't think I would explain it to them, but I'd just explain you have to try your best. I know I get a lot of kids that are like, "I don't know how to do this" I had one who was emotionally sensitive and he was very insecure about himself and he would just throw a fit. But one thing that I keep telling them is that I would want them to try their best first and then if they're still struggling and still frustrated, like then what can you do to fix it or what can you do to improve in something. I don't

	make it into they solve your own problem, but me just guiding them instead of helping them to make them more independent. Especially with kindergarteners, they're kind of used to having mom and dad by them all the time and so I kind of had to guide them to think of how can they solve their own problems, how can they think more positive on themselves to solve a problem rather than thinking that they can't do it all the time. And then just making, having positive feedback. FI1-4 F - We haven't really talked the title called growth mindset but the very first math class, when a student really didn't understand how to do the workbook pages, the assignment that I'm giving them I'm teaching them just do your best. There's no wrong or right, but if you do your best to show me that you're trying really hard, that's good enough. I'm just giving them, "In Mrs. O's classroom, what I want to see is just do your best." So I think that just lines up with growth mindset. DI1-4 "D - Yeah and you know what, that is kind of part of the environment is creating the kind of environment. I'd rather see a kid who is working, working, working past obstacles. Oh, that didn't work, I'm going to try something else. Whose got that grit. That person to me is showing way more growth than a person who sits down and gets it right and they're done. So training your students that...I tell my students that the harder it is that's your brain getting stronger, you know. If we're doing easy things all the time, your brain isn't getting stronger. I tell them their brain is growing. To teach them what to do when something doesn't work, or when an answer isn't right. Being able to get over that obstacle and embrace that as part of the journey. And setting high goals for your kids and high standards.
Misc. Evidence of growth mindset at start of year	DI1-4 "K - Do you ever hear any of them say stuff to each other? D - Yeah. It's funny we have our social emotional learning. They learn a lot about self-talk and I hear them talk to themselves sometimes which is really sweet. " FI1-10 Is there anything that you thought of while we were talking about your class now or your methods course that popped in your head, your memory? F - the growth mindset. I was just casually talking about my expectation in math. Just do your best. I never connect that with ok this is a growth mindset. I will have to talk about growth mindset.That is something that I can do with my students tomorrow because we don't have tier tomorrow. Maybe that extra 30 minutes we can talk a little more about it. CI1-4 K - Ok. Did that positivity go towards the math too? C - Yes, I remember the teacher I had, she was a graduate student I think. She was working on her, well I don't know. She understood that school is, balancing school and life was tricky. She would tell us her expectations. Then she trusted us enough and she would encourage us. She always had positive feedback. She was helpful. K - So what happened when, if somebody didn't get the math or was frustrated? C - She would even say you can come in and meet with her during office hours. She always says her office hours are from this time and this time, and you guys are free to come in. She replied to her emails. They were always positive and not sounding like it was a burden to email her. She was just very open. All of the teachers I had were very open, understanding, and answering the emails and replying. They always included their office hours. Even if you couldn't make it to their office hours, they were flexible about it.